CHAPTER 4

Linear Functions

Section 4A

Characteristics of Linear Functions

- 4-1 Identifying Linear Functions
- 4-2 Using Intercepts
- Connecting Algebra to Geometry Area in the Coordinate Plane
- 4-3 Rate of Change and Slope
- Algebra Lab Explore Constant Changes
- 4-4 The Slope Formula
- 4-5 Direct Variation

Section 4B

Using Linear Functions

- 4-6 Slope-Intercept Form
- 4-7 Point-Slope Form
- Technology Lab Graph Linear Functions
- Connecting Algebra to Data Analysis Interpreting Trend Lines
- 4-8 Line of Best Fit
- 4-9 Slopes of Parallel and Perpendicular Lines
- Technology Lab The Family of Linear Functions
- 4-10 Transforming Linear Functions
- Extension Absolute-Value Functions

Pacing Guide for 45-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 4

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
4-1 Lesson	4-2 Lesson	Connecting Algebra to Geometry	4-3 Lesson	Algebra Lab
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
4-4 Lesson	4-5 Lesson	Multi-Step Test Prep Ready to Go On?	4-6 Lesson	4-7 Lesson
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
Technology Lab	Connecting Algebra to Data Analysis	4-8 Lesson	4-8 Lesson	4-9 Lesson
DAY 16	**DAY 17**	**DAY 18**	**DAY 19**	**DAY 20**
Technology Lab	4-10 Lesson	Multi-Step Test Prep Ready to Go On?	Extension	Chapter Review
DAY 21				
Chapter Test				

Pacing Guide for 90-Minute Classes

Teacher One Stop™ Calendar Planner®

Chapter 4

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
4-1 Lesson 4-2 Lesson	Connecting Algebra to Geometry 4-3 Lesson	Algebra Lab 4-4 Lesson	4-5 Lesson Multi-Step Test Prep Ready to Go On?	4-6 Lesson 4-7 Lesson
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	
Technology Lab Connecting Algebra to Data Analysis	4-8 Lesson 4-9 Lesson	Technology Lab 4-10 Lesson	Multi-Step Test Prep Ready to Go On? Extension	Chapter Review Chapter Test

ONGOING ASSESSMENT and INTERVENTION

	DIAGNOSE	PRESCRIBE
Assess Prior Knowledge	**Before the Chapter**	
	Diagnose readiness for the chapter.	Prescribe intervention.
	Are You Ready? SE	***Are You Ready? Intervention***
Formative Assessment	**Before Every Lesson**	
	Diagnose readiness for the lesson.	Prescribe intervention.
	Warm Up TE	**Reteach** CRB
	During Every Lesson	
	Diagnose understanding of lesson concepts.	Prescribe intervention.
	Check It Out! SE	**Reading Strategies** CRB
	Questioning Strategies TE	***Success for Every Learner***
	Think and Discuss SE	***Lesson Tutorial Videos***
	Write About It SE	
	Journal TE	
	After Every Lesson	
	Diagnose mastery of lesson concepts.	Prescribe intervention.
	Lesson Quiz TE	**Reteach** CRB
	Test Prep SE	**Test Prep Doctor** TE
	ExamView™ Assessment Suite	***Homework Help* Online**
	Before Chapter Testing	
	Diagnose mastery of concepts in chapter.	Prescribe intervention.
	Ready to Go On? SE	***Ready to Go On? Intervention***
	Multi-Step Test Prep SE	**Scaffolding Questions** TE
	Section Quizzes AR	**Reteach** CRB
	ExamView™ Assessment Suite	***Lesson Tutorial Videos***
	Before High Stakes Testing	
	Diagnose mastery of benchmark concepts.	Prescribe intervention.
	College Entrance Exam Practice SE	***College Entrance Exam Practice***
	Standardized Test Prep SE	
Summative Assessment	**After the Chapter**	
	Check mastery of chapter concepts.	Prescribe intervention.
	Multiple-Choice Tests (Forms A, B, C)	**Reteach** CRB
	Free-Response Tests (Forms A, B, C)	***Lesson Tutorial Videos***
	Performance Assessment AR	
	Cumulative Test AR	
	ExamView™ Assessment Suite	

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* **CRB** = *Chapter Resource Book* **AR** = *Assessment Resources* Available online

CHAPTER 4

Lesson Resources

Before the Lesson

Prepare

Teacher One Stop
- Editable lesson plans
- Calendar Planner
- Easy access to all chapter resources

Lesson Transparencies
- Teacher Tools

Teach the Lesson

Introduce

Alternate Openers: Explorations

Lesson Transparencies
- Warm Up
- Problem of the Day

Teach

Lesson Transparencies
- Teaching Transparencies

Know-It Notebook™
- Vocabulary
- Key Concepts

Power Presentations

Lesson Tutorial Videos SPANISH

Interactive Online Edition
- Lesson Activities
- Lesson Tutorial Videos

Lab Activities

Lab Resources Online

Online Interactivities SPANISH

TechKeys

Practice the Lesson

Practice

Chapter Resources
- Practice A, B, C

Practice and Problem Solving Workbook

IDEA Works!® Modified Worksheets and Tests

ExamView™ Assessment Suite

Homework Help Online

Online Interactivities SPANISH

Interactive Online Edition
- Homework Help

Apply

Chapter Resources
- Problem Solving

Practice and Problem Solving Workbook

Interactive Online Edition
- Chapter Project

Project Teacher Support

After the Lesson

Reteach

Chapter Resources
- Reteach
- Reading Strategies ELL

Success for Every Learner

Review

Interactive Answers and Solutions

Solutions Key

Know-It Notebook™
- Big Ideas
- Chapter Review

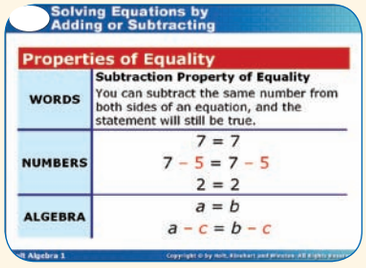
Extend

Chapter Resources
- Challenge

Technology Highlights for the Teacher

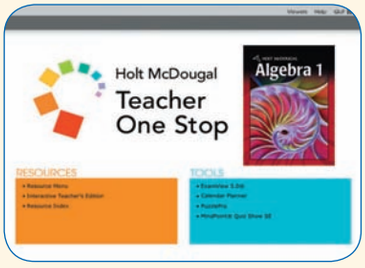
Power Presentations

Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in this chapter.



Teacher One Stop SPANISH

Easy access to the chapter resources and assessments. Includes lesson planning software.



Interactive Online Edition

This chapter includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.

KEY: **SE** = *Student Edition* **TE** = *Teacher's Edition* ELL English Language Learners SPANISH Spanish version available Available online

CHAPTER 4

Reaching All Learners

Teaching tips to help all learners appear throughout the chapter. A few that target specific students are included in the lists below.

All Learners

Lab Activities
Practice and Problem Solving Workbook
Know-It Notebook

Special Needs Students

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Inclusion ..... TE
IDEA Works!® Modified Worksheets and Tests
Ready to Go On? Intervention
Know-It Notebook
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

Developing Learners

Practice A ..... CRB
Reteach ..... CRB
Reading Strategies ..... CRB
Are You Ready? ..... SE
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH
Lesson Tutorial Videos SPANISH

On-Level Learners

Practice B ..... CRB
Problem Solving ..... CRB
Vocabulary Connections ..... SE
Questioning Strategies ..... TE
Ready to Go On? Intervention
Know-It Notebook
Homework Help Online
Online Interactivities SPANISH

Advanced Learners

Practice C ..... CRB
Challenge ..... CRB
Challenge Exercises ..... SE
Reading and Writing Math *Extend* ..... TE
Are You Ready? Enrichment
Ready To Go On? Enrichment

English Language Learners

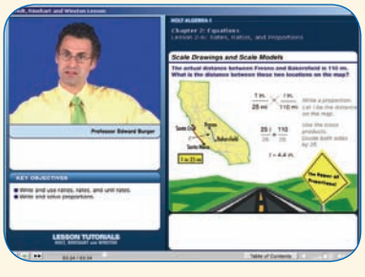
ENGLISH LANGUAGE LEARNERS

Reading Strategies ..... CRB
Are You Ready? Vocabulary ..... SE
Vocabulary Connections ..... SE
Vocabulary Review ..... SE
English Language Learners ..... TE
Success for Every Learner
Know-It Notebook
Multilingual Glossary
Lesson Tutorial Videos SPANISH

Technology Highlights for Reaching All Learners

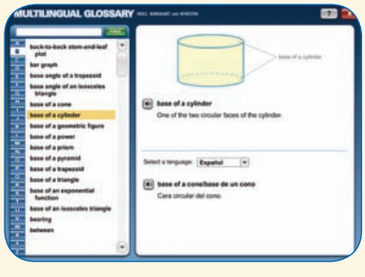
Lesson Tutorial Videos SPANISH

Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in this chapter.



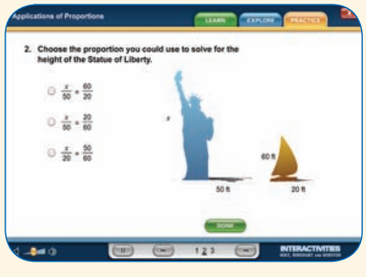
Multilingual Glossary

Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and other languages.



Online Interactivities

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



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CHAPTER 4

Ongoing Assessment

Assessing Prior Knowledge

Determine whether students have the prerequisite concepts and skills for success in the chapter.

Are You Ready? ..... SE
Warm Up ..... TE

Test Preparation

Provide review and practice for the chapter and standardized tests.

Multi-Step Test Prep ..... SE
Study Guide: Review ..... SE
Test Tackler ..... SE
Standardized Test Prep ..... SE
College Entrance Exam Practice ..... SE
Countdown to Mastery ..... SE
IDEA Works!® Modified Worksheets and Tests

Alternative Assessment

Assess students' understanding of the chapter concepts and combined problem-solving skills.

Chapter Project ..... SE
Alternative Assessment ..... TE
Performance Assessment ..... AR
Portfolio Assessment ..... AR

Lesson Assessment

Provide formative assessment for each lesson of the chapter.

Questioning Strategies ..... TE
Think and Discuss ..... SE
Check It Out! Exercises ..... SE
Write About It ..... SE
Journal ..... TE
Lesson Quiz ..... TE
Alternative Assessment ..... TE
IDEA Works!® Modified Worksheets and Tests

Weekly Assessment

Provide formative assessment for each section of the chapter.

Multi-Step Test Prep ..... SE
Ready to Go On? ..... SE
Section Quizzes ..... AR
ExamView™ Assessment Suite

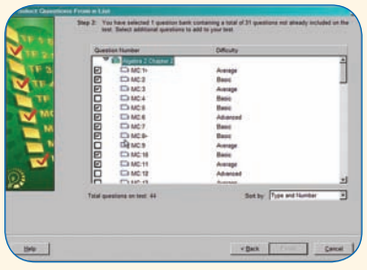
Chapter Assessment

Provide summative assessment of the chapter mastery.

Chapter Test ..... SE
Chapter Test (Levels A, B, C) ..... AR
• Multiple Choice • Free Response
Cumulative Test ..... AR
ExamView™ Assessment Suite
IDEA Works!® Modified Worksheets and Tests

Technology Highlights for Assessment

ExamView™ Assessment Suite
Use the chapter problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.



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CHAPTER 4

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests, along with a performance assessment, are available in the *Assessment Resources.*

A Chapter Test

C Chapter Test

MULTIPLE CHOICE

B Chapter Test

Select the best answer.

1. Which set of ordered pairs satisfies a linear function?

A

x	1	1	1	1
y	2	4	6	8

B

x	1	2	3	4
y	2	4	8	16

C

x	–1	1	3	5
y	8	6	4	2

D

x	–3	–2	–1	0
y	2	–2	4	–4

2. A vacation home in Orlando, Florida, rents for $105 per day. The function $f(x) = 105x$ gives the cost of renting the home for x days. What is the domain of this function?

F $x \geq 0$

G {0, 1, 2, 3, ...}

H {0, 105, 210, 315, ...}

J all real numbers

3. A parking meter gives 30 minutes for each quarter and 6 minutes for each nickel. The equation $30x + 6y = 60$ describes the number of quarters x and nickels y that you need to park for 60 minutes. What does the x-intercept represent?

A You need 2 quarters and no nickels to park for 60 minutes.

B You need 10 nickels and no quarters to park for 60 minutes.

C You need 6 nickels and no quarters to park for 60 minutes.

D You need 30 quarters and no nickels to park for 60 minutes.

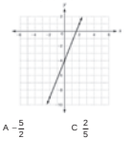
4. This table shows the U.S. federal minimum hourly wage in different years. During which time interval did the wage increase at the greatest rate?

Year	1979	1980	1981	1990	1991
Wage ($)	2.90	3.10	3.35	3.80	4.25

F 1979 to 1980 H 1981 to 1990

G 1980 to 1981 J 1990 to 1991

5. Find the slope of this line.



A $-\frac{5}{2}$ C $\frac{2}{5}$

B $-\frac{2}{5}$ D $\frac{5}{2}$

6. Find the slope of the line that contains the points (1, –1) and (–2, 8).

F –5 H $-\frac{7}{3}$

G –3 J $-\frac{1}{3}$

7. Which equation is NOT a direct variation?

A $y = 50x$ C $-2y = x$

B $5x + 2y = 10$ D $-3x + 2y = 0$

8. Which equation describes the line with a slope of 5 and y-intercept of –3?

F $y = -3x + 5$ H $y = 5x - 3$

G $y = 3x - 5$ J $y = 5x + 3$

B Chapter Test *(continued)*

9. What is the slope and y-intercept of the graph?

A slope $= -\frac{1}{4}$, y-intercept $= 2$

B slope $= -\frac{1}{4}$, y-intercept $= 3$

C slope $= 4$, y-intercept $= 4$

D slope $= 4$, y-intercept $= -4$

10. Which equation describes the line with a slope of 2 that contains the point (4, –3)?

F $y - 4 = 2(x + 3)$ H $y + 3 = 2(x - 4)$

G $2(y - 3) = x + 4$ J $2(y + 4) = x + 3$

11. The table shows the age of several people and how many hours a day they spend playing computer and/or video games.

Age (yrs)	15	63	33	22	15
Playing (hrs/day)	5	0	2	8	0

Which equation could represent a line of best fit for this data?

A $y = 53x - 7.9$

B $y = -7.9x + 53$

C $y = 5.3x - 0.079$

D $y = -0.079x + 5.3$

12. Here are four linear equations.

I $4x + 3y = 15$ II $3x - 4y = -8$

III $y + 1 = \frac{4}{3}(x - 6)$ IV $y = \frac{3}{4}x - 5$

Which pair of lines are parallel?

F I and II H II and IV

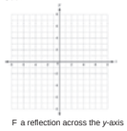
G I and III J III and IV

13. Which equation describes a line that passes through (–6, 8) and is perpendicular to the line described by $y = 2x - 4$?

A $y = -2x - 4$ C $y = \frac{1}{2}x + 11$

B $y = -\frac{1}{2}x + 5$ D $y = 2x + 20$

14. Graph $f(x) = -3x + 4$ and $g(x) = 3x + 4$. Which describes the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$?



F a reflection across the y-axis

G a reflection across the y-axis and a translation 4 units up

H a rotation (less steep) about (0, 4)

J a translation up 6 units

FREE RESPONSE

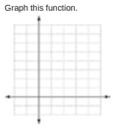
B Chapter Test

1. Do these ordered pairs satisfy a linear function? Explain.

x	1	1	1	1
y	–5	0	5	10

2. At a convenience store, bottles of water cost $1.20. The function $f(x) = 1.2x$ gives the cost of buying x bottles.

Graph this function.



Give its domain and range.

domain: _____

range: _____

3. A recycling center pays 2 cents per aluminum can and 5 cents per glass bottle. The equation $2x + 5y = 100$ describes the number of cans x and bottles y that you need to earn $1.00. What is the value of each intercept, and what does each represent?

x-intercept: _____

_____

y-intercept: _____

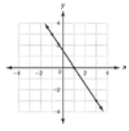
_____

4. This table shows the median age of women who were first married in different years. During which time interval did the age increase at the greatest rate? What was the rate of increase for that interval?

Year	1950	1980	1990	1995	2000
Age (yr)	20.3	22.0	23.9	24.5	25.1

_____

5. Find the slope of this line.



_____

6. Find the slope of the line that contains the points (–6, 4) and (2, –8).

_____

7. Tell whether the equation $3x - 6y = 0$ is a direct variation. Explain. If the equation is a direct variation, identify the constant of variation.

_____

_____

_____

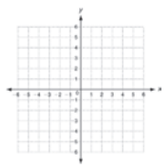
_____

B Chapter Test *(continued)*

8. Write an equation in slope-intercept form that describes the line with a slope of $\frac{1}{5}$ and y-intercept of –10.

_____

9. Graph the line described by $y = \frac{5}{2}x - 4$.



10. An artisan builds birdhouses out of pine for a craft fair. The table shows the relationship between the number of hours the artisan spends on a birdhouse, and how much she charges for it.

Hours	2	2	6	4	8
Price (dollars)	35	45	70	65	75

Find an equation for a line of best fit.

_____

11. Write an equation in point-slope form that describes the line with a slope of $\frac{3}{5}$ that contains the point (–2, 6).

_____

12. Identify which lines are parallel.

I $y = 4x$ II $y = -\frac{1}{4}x - 3$

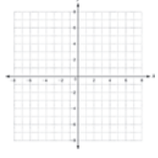
III $12x - 3y = 6$ IV $4(y + 6) = x - 3$

_____

13. Write an equation in slope-intercept form for the line that passes through (–3, 2) and is perpendicular to the line described by $y = \frac{3}{2}x + 4$.

_____

14. Graph $f(x) = x - 1$ and $g(x) = 5x - 1$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.



_____

_____

PERFORMANCE ASSESSMENT

Chapter Test

Automobile Trends

Purpose

This performance task assesses the student's ability to write the equation of a trend line in slope-intercept form or point-slope form.

Time

40–55 minutes

Grouping

Partners

Preparation Hints

Review the concept of a *trend line*. Remind students that they used the graph of a trend line to make predictions. Now that students can write the equation of a line, they can also use the equation to make predictions.

Overview

Students graph a trend line that they feel fits a scatter plot well. Depending upon whether the y-intercept is visible, they use either slope-intercept form or point-slope form to write the equation of the trend line. Lastly, they use the trend line to make predictions (both interpolation and extrapolation).

Introduce the Task

Ask what students know about automobile fuel economy. Some students may have heard the terms *MPG* or *mileage* to describe miles per gallon, or heard their parents discussing fuel economy when purchasing a new car. Ask whether students know what types of automobiles get better or poorer mileage, and what types of factors influence fuel economy. Lastly, ask whether any students know about (or even own) a hybrid automobile.

Performance Indicators

____ Draws a line that appropriately summarizes the trend of linear data.

____ Identifies or calculates features of the line necessary to write a linear equation (slope, y-intercept, or collinear points).

____ Writes the equation of the trend line in slope-intercept or point-slope form.

____ Uses the trend line's equation to make predictions.

Scoring Rubric

Level 4: Student solves all problems correctly and gives explanations.

Level 3: Student solves all problems correctly but gives no explanations.

Level 2: Student solves some problems but gives no explanations.

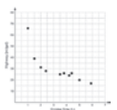
Level 1: Student is not able to solve any of the problems.

Chapter Test *(continued)*

Automobile Trends

For 1–6, use this data about engine size in liters x and highway fuel economy in miles per gallon y for a few model-year 2005 automobiles.

Engine size (L)	1.0	1.5	2.0	2.4	3.5	3.8	4.2	4.4	5.0	5.9
Highway (mi/gal)	66	39	31	28	25	26	24	26	20	17



1. Discuss with your partner what a good trend line for this data would look like. Using a ruler, add a trend line to the scatter plot. Make the line intersect the y-axis.

2. Estimate the y-intercept of your line. _____

3. Estimate the slope of your line. Explain how you did this. _____

_____

4. Write the equation of your line in slope-intercept form. _____

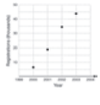
5. Use your equation to estimate the highway fuel economy of an automobile with a 3.0 L engine. _____

6. One data point represents a hybrid automobile that uses both electricity and gasoline fuel. Which point do you think is the hybrid? How is that data point different from the rest? _____

_____

For 7–9, use this data about the number of new hybrid automobile registrations y in each year x in the U.S.

Year	2000	2001	2002	2003
Registrations (thousands)	6.5	19.0	34.5	43.4



7. Draw a trend line that passes through two of the data points. Explain why you choose those two points.

_____

8. Because the x-axis doesn't start at zero, you can't see the y-intercept. So, use the two points you chose in question 7 to write the equation of the line in point-slope form. _____

9. Assuming the linear trend continues, use your equation to estimate the number of new registrations in 2007. _____

CHAPTER 4

MATHEMATICAL PRACTICES **Reason abstractly and quantitatively.**

SECTION 4A

Characteristics of Linear Functions

MULTI-STEP TEST PREP Students write, graph, and evaluate algebraic expressions to describe maximum heart rates during exercise.

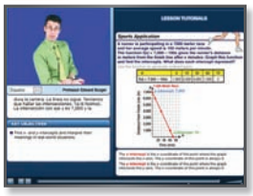
SECTION 4B

Using Linear Functions

MULTI-STEP TEST PREP Students use linear equations to model and solve problems involving walking at constant speeds. They describe how changing slope affects the graph of this function.

Interactivities Online ▶

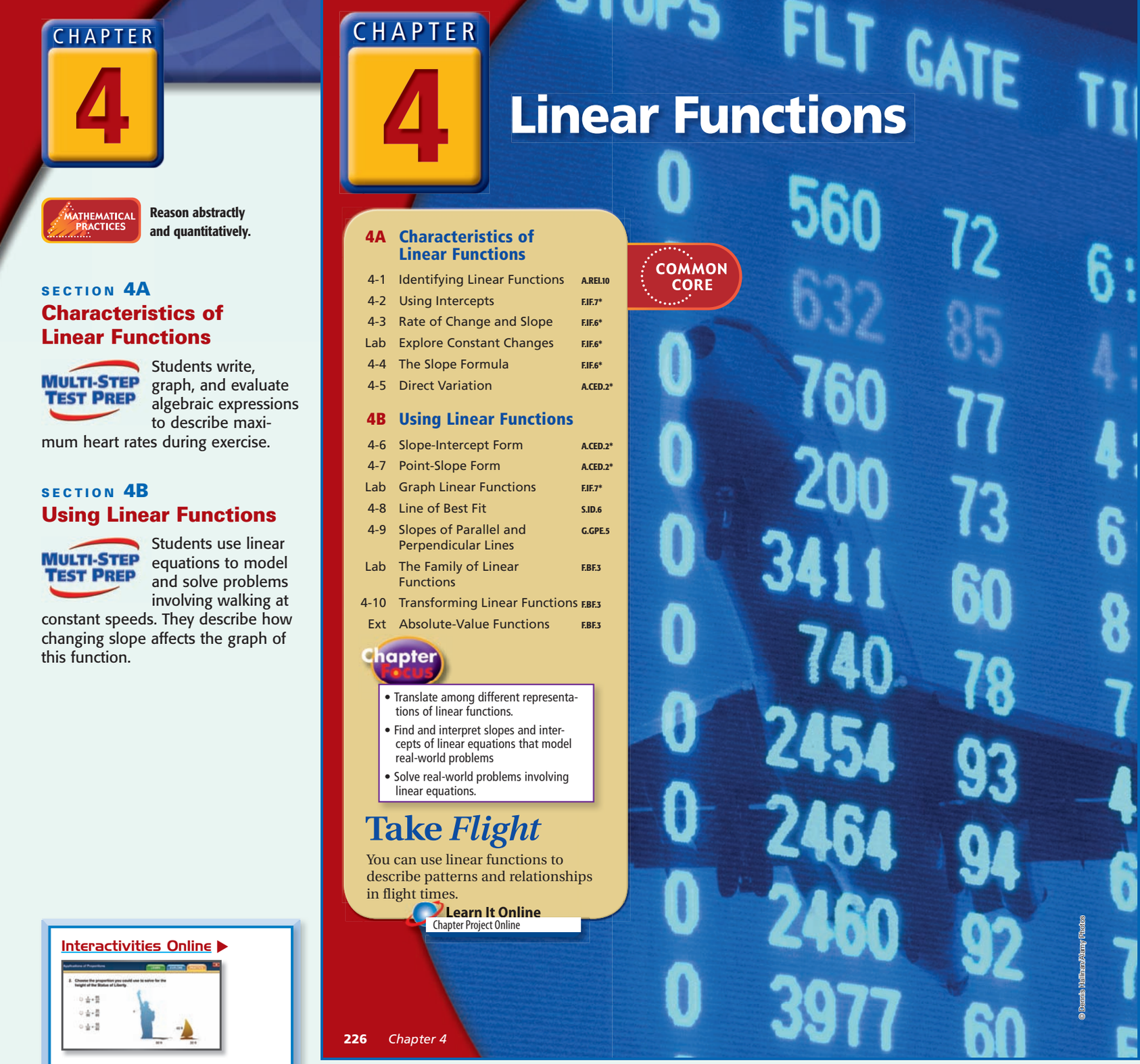
Video Lesson Tutorials Online



Lesson Tutorial Videos are available for EVERY example.

CHAPTER 4

Linear Functions



COMMON CORE

4A Characteristics of Linear Functions

4-1	Identifying Linear Functions	A.REI.10
4-2	Using Intercepts	F.IF.7*
4-3	Rate of Change and Slope	F.IF.6*
Lab	Explore Constant Changes	F.IF.6*
4-4	The Slope Formula	F.IF.6*
4-5	Direct Variation	A.CED.2*

4B Using Linear Functions

4-6	Slope-Intercept Form	A.CED.2*
4-7	Point-Slope Form	A.CED.2*
Lab	Graph Linear Functions	F.IF.7*
4-8	Line of Best Fit	S.ID.6
4-9	Slopes of Parallel and Perpendicular Lines	G.GPE.5
Lab	The Family of Linear Functions	F.BF.3
4-10	Transforming Linear Functions	F.BF.3
Ext	Absolute-Value Functions	F.BF.3

Chapter Focus

- Translate among different representations of linear functions.
- Find and interpret slopes and intercepts of linear equations that model real-world problems
- Solve real-world problems involving linear equations.

Take *Flight*

You can use linear functions to describe patterns and relationships in flight times.

Learn It Online
Chapter Project Online

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Chapter Project

A project for this chapter is available online.

Project Resources

All project resources for teachers and students are provided online.

Learn It Online
Chapter Project Online

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. coefficient **E**
2. coordinate plane **C**
3. transformation **A**
4. perpendicular **B**

A. a change in the size or position of a figure

B. forming right angles

C. a two-dimensional system formed by the intersection of a horizontal number line and a vertical number line

D. an ordered pair of numbers that gives the location of a point

E. a number that is multiplied by a variable

Ordered Pairs

Graph each point on the same coordinate plane.

5. $A(2, 5)$
6. $B(-1, -3)$
7. $C(-5, 2)$
8. $D(4, -4)$
9. $E(-2, 0)$
10. $F(0, 3)$
11. $G(8, 7)$
12. $H(-8, -7)$

Solve for a Variable

Solve each equation for the indicated variable.

13. $2x + y = 8$; y **$y = 8 - 2x$**
14. $5y = 5x - 10$; y **$y = x - 2$**
15. $2y = 6x - 8$; y **$y = 3x - 4$**
16. $10x + 25 = 5y$; y **$y = 2x + 5$**

Evaluate Expressions

Evaluate each expression for the given value of the variable.

17. $4g - 3$; $g = -2$ **-11**
18. $8p - 12$; $p = 4$ **20**
19. $4x + 8$; $x = -2$ **0**
20. $-5t - 15$; $t = 1$ **-20**

Connect Words and Algebra

21. The value of a stock begins at \$0.05 and increases by \$0.01 each month. Write an equation representing the value of the stock v in any month m. **$v = 0.05 + 0.01m$**
22. Write a situation that could be modeled by the equation $b = 100 - s$.
Possible answer: The amount of money in your bank account equals \$100 minus the amount spent.

Rates and Unit Rates

Find each unit rate.

23. 322 miles on 14 gallons of gas **23 mi/gal**
24. \$14.25 for 3 pounds of deli meat **\$4.75/lb**
25. 32 grams of fat in 4 servings **8 g/serving**
26. 120 pictures on 5 rolls of film **24 pictures/roll**

CHAPTER 4

ARE YOU READY?

Organizer

Objective: Assess students' understanding of prerequisite skills.

Assessing Prior Knowledge

INTERVENTION

Diagnose and Prescribe

Use this page to determine whether intervention is necessary or whether enrichment is appropriate.

Resources

***Are You Ready? Intervention and Enrichment* Worksheets**

***Are You Ready?* Online**

Answers

5–12. See Additional Answers.

ARE YOU READY?
Diagnose and Prescribe

NO INTERVENE

Prerequisite Skill	*Are You Ready? Intervention*, Chapter 4 Worksheets	*Are You Ready? Intervention*, Chapter 4 Online	
Ordered Pairs	Skill 79	Activity 79	Diagnose and Prescribe Online
Solve for a Variable	Skill 72	Activity 72	
Evaluate Expressions	Skill 60	Activity 60	
Connect Words and Algebra	Skill 58	Activity 58	
Rates and Unit Rates	Skill 13	Activity 13	

YES ENRICH

Are You Ready? Enrichment, Chapter 4
Worksheets
Online

Organizer

Objective: Help students organize the new concepts they will learn in this chapter.

Online Edition
Multilingual Glossary

Resources

Learn It Online
Multilingual Glossary

Possible answers to *Vocabulary Connections*

1. a line
2. where a graph intersects the *x*-axis
3. a slant or an incline
4. functions that are somehow related or have similar characteristics

CHAPTER 4 Study Guide: Preview

Where You've Been

Previously, you

- wrote equations in function notation.
- graphed functions.
- identified the domain and range of functions.
- identified independent and dependent variables.

In This Chapter

You will study

- writing and graphing linear functions.
- identifying and interpreting the components of linear graphs, including the *x*-intercept, *y*-intercept, and slope.
- graphing and analyzing families of functions.

Where You're Going

You can use the skills in this chapter

- to solve systems of linear equations in Chapter 5.
- to identify rates of change in linear data in biology and economics.
- to make calculations and comparisons in your personal finances.

Key Vocabulary/Vocabulario

constant of variation	constante de variación
direct variation	variación directa
family of functions	familia de funciones
linear function	función lineal
parallel lines	líneas paralelas
perpendicular lines	líneas perpendiculares
slope	pendiente
transformation	transformación
x-intercept	intersección con el eje *x*
y-intercept	intersección con el eje *y*

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

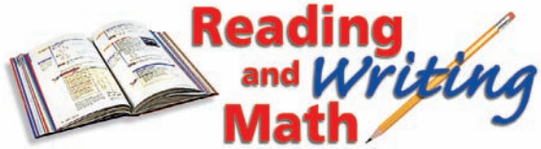
1. What shape do you think is formed when a **linear function** is graphed on a coordinate plane?
2. The meaning of *intercept* is similar to the meaning of *intersection*. What do you think an ***x*-intercept** might be?
3. **Slope** is a word used in everyday life, as well as in mathematics. What is your understanding of the word *slope*?
4. A family is a group of related people. Use this concept to define **family of functions**.

MATHEMATICAL PRACTICES

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. **Make sense of problems and persevere in solving them.** Pages 234–236, 240–242, 266, 278, 308
2. **Reason abstractly and quantitatively.** Pages 233, 237, 238, 278
3. **Construct viable arguments and critique the reasoning of others.** Pages 233, 235–236, 239, 241–242, 247, 257, 263, 271, 279, 289, 296, 304
4. **Model with mathematics.** Pages 256, 257–285, 262, 271, 304
5. **Use appropriate tools strategically.** Pages 240–241, 283, 300
6. **Attend to precision.** Pages 241, 250, 259, 266, 281, 285, 298
7. **Look for and make use of structure.** Pages 234–236
8. **Look for and express regularity in repeated reasoning.** Pages 252

228 Chapter 4



CHAPTER 4

Study Strategy: Use Multiple Representations

Representing a math concept in more than one way can help you understand it more clearly. As you read the explanations and example problems in your text, note the use of tables, lists, graphs, diagrams, and symbols, as well as words to explain a concept.

In this example, the given function is described using an equation, a table, ordered pairs, and a graph.

Graphing Functions

Graph each function.

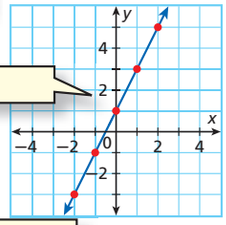
A $2x + 1 = y$ — Equation

Step 1 Choose several values of x and generate ordered pairs. **Step 2** Plot enough points to see a pattern.

Table

x	$2x + 1 = y$	(x, y)
-3	$2(-3) + 1 = -5$	$(-3, -5)$
-2	$2(-2) + 1 = -3$	$(-2, -3)$
-1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$
3	$2(3) + 1 = 7$	$(3, 7)$

Graph



Ordered Pairs

Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.

Try This

1. If an employee earns \$8.00 an hour, $y = 8x$ gives the total pay y the employee will earn for working x hours. For this equation, make a table of ordered pairs and a graph. Explain the relationships between the equation, the table, and the graph. How does each one describe the situation?
2. What situations might make one representation more useful than another?

COMMON CORE **Standards for Mathematical Content** High School

		LESSON
CREATING EQUATIONS Create equations that describe numbers of relationships		
CC.9-12.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	4-1, 4-2, 4-5, 4-6, 4-7
REASONING WITH EQUATIONS AND INEQUALITIES Represent and solve equations and inequalities graphically		
CC.9-12.A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	4-1
INTERPRETING FUNCTIONS Interpret functions that arise in applications in terms of the context		
CC.9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	4-3, Lab 4-3, 4-4, 4-6
INTERPRETING FUNCTIONS Analyze functions using different representations.		
CC.9-12.F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	4-1, 4-2, 4-5, 4-6, 4-7, TLab 4-7, 4-9

Organizer

Objective: Help students apply strategies to understand and retain key concepts.

INTERACTIVE **Online Edition**

Study Strategy: Use Multiple Representations

Discuss Students make key mathematical connections when they see the same concept presented in various formats side by side.

For the example presented, the numbers in the domain of $2x + 1 = y$ appear in green and are also the x-coordinates of the points plotted on the graph. The numbers in the range appear in blue and are also the y-coordinates of the points plotted on the graph.

Extend As students work through the application problems in this chapter, have them discuss which representation they think is best for that problem and why.

Answers to *Try This*

1–2. See Additional Answers.

Reading Connection

Strength in Numbers
by Sherman K. Stein

This is a careful, beautifully written book covering topics from arithmetic to calculus. Stein expounds on each in a way that will enlighten algebra students and delight their teachers. He continually reinforces the important idea that the more mathematics a person knows, the more career options are available to him or her.

Activity After students study the slope of a line, direct them to, Chapter 28 *How Steep is a Curve?* Ask them to explain how a curve can have a slope and how it can be measured. Then ask: How is the slope of a curve like the slope of a line? How is it different?

SECTION 4A

Linear Functions

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 4-1 Identifying Linear Functions • Identify linear functions and linear equations. • Graph linear functions that represent real-world situations and give their domain and range. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Optional** graphing calculator
Lesson 4-2 Using Intercepts • Find *x*- and *y*-intercepts and interpret their meanings in real-world situations. • Use *x*- and *y*-intercepts to graph lines. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Lesson 4-3 Rate of Change and Slope • Find rates of change and slopes. • Relate a constant rate of change to the slope of a line. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Technology Lab	
Algebra Lab Explore Constant Changes • Explore the relationship between constant change and the slope of a line. ☑ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Lab Recording Sheet	
Lesson 4-4 The Slope Formula • Find slope by using the slope formula. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** number cubes (MK)
Lesson 4-5 Direct Variation • Identify, write, and graph direct variation. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** ruler (MK), graphing calculator

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook.

Lesson 1: 1, 3, 5, 7, 9, 11, 15, 17, 19, 27, 29, 31, 33, 35, 37, 39, 41, 55, 57, 63

Lesson 2: 1, 3, 5, 7, 13, 15, 17, 19, 21, 35, 37, 41, 47

Lesson 3: 1, 5, 7, 9, 11, 15, 17, 19, 23, 29, 31

Lesson 4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23a, 23b, 23c, 25a, 27, 29, 31, 33, 35, 37

Lesson 5: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 29, 33, 41, 43

All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Math Background

COMMON CORE PROFESSIONAL DEVELOPMENT

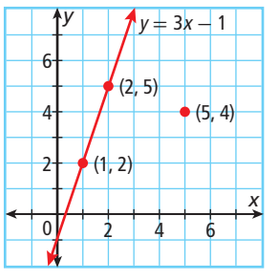
LINEAR FUNCTIONS

Lesson 4-1

A solution of a linear equation in two variables, x and y, is an ordered pair, (a, b), such that when a is substituted for x and b is substituted for y the resulting equation is true. Thus the ordered pair $(2, 5)$ is a solution of $y = 3x - 1$ because $5 = 3(2) - 1$ is a true statement. The ordered pair $(5, 4)$ is not a solution of the equation because $4 \neq 3(5) - 1$.

In contrast to linear equations in one variable, linear equations in two variables have infinitely many solutions. It is important for students to understand the connection between the solutions of a linear equation and the graph of the equation—the graph of an equation is precisely the set of ordered pairs that are solutions of the equation.

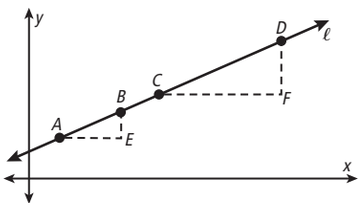
A specific example helps to clarify this last point. The graph of $y = 3x - 1$ consists of all ordered pairs (a, b) such that $b = 3a - 1$. In other words, the graph consists of all ordered pairs of the form $(a, 3a - 1)$. As shown above, the point $(2, 5)$ satisfies this relationship and so it must lie on the graph. Conversely, the point $(1, 2)$ lies on the graph, so it must be a solution of the equation, and indeed $2 = 3(1) - 1$. It was also shown above that $(5, 4)$ is *not* a solution of the equation; therefore this point does *not* lie on the graph.



SLOPE

Lessons 4-3, 4-4

The slope of a line is defined as the ratio of rise to run for *any pair* of points on the line.



To see why the slope of line ℓ is the same whether points A and B are used or points C and D are used, draw $\overline{AE}$ and $\overline{CF}$ parallel to the x-axis, and $\overline{BE}$ and $\overline{DF}$ parallel to the y-axis. Then $\overline{AE}$ and $\overline{CF}$ are parallel (since they are both parallel to the x-axis) and $\angle A \cong \angle C$, since they are corresponding angles formed by parallel lines cut by a transversal. By a similar argument, $\angle B \cong \angle D$. Thus, $\triangle ABE$ is similar to $\triangle CDF$. Corresponding sides of similar triangles are proportional so $\frac{BE}{DF} = \frac{AE}{CF}$, which can be rewritten as $\frac{BE}{AE} = \frac{DF}{CF}$. This shows that the ratio of rise to run (slope) is the same for both pairs of points.

DIRECT VARIATION

Lesson 4-5

A *direct variation* is a linear relationship that can be written in the form $y = kx$, where k is a nonzero constant. A direct variation is a proportional relationship because the ratio $\frac{y}{x}$ is constant for every ordered pair for which $x \neq 0$.

There are two ways in which a function may fail to represent a direct variation. The first category includes linear functions with a nonzero constant ($y = mx + b$, where $b \neq 0$). These functions have constant slope, but the ratio $\frac{y}{x}$ is not constant for every ordered pair. Therefore, they are neither proportional nor direct variations.

The second category includes functions whose slope may vary between different pairs of points, in other words, nonlinear functions. The varying slope of nonlinear functions is the basis of differential calculus, which students may study in future classes.

4-1 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Identify linear functions and linear equations.

Graph linear functions that represent real-world situations and give their domain and range.

Algebra Lab
In *Algebra Lab Activities*

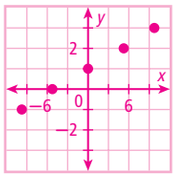
Online Edition
Tutorial Videos, Graphing Calculator

Countdown

Power Presentations with PowerPoint®

Warm Up

1. Solve $2x - 3y = 12$ for y.
$y = \frac{2}{3}x - 4$
2. Graph $y = \frac{1}{5}x + 1$ for D: {−10, −5, 0, 5, 10}.



Also available online

Math Humor

Q: How are linear functions similar to Cupid?

A: They have a partner for every number.

COMMON CORE

CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Learn It Online
State Resources Online

COMMON CORE

4-1 Identifying Linear Functions

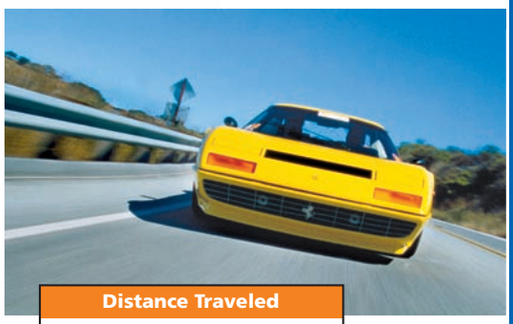
CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.… *Also* **CC.9-12.F.IF.7***, **CC.9-12.A.CED.2***, **CC.9-12.F.IF.5***, **CC.9-12.F.LE.2***

Objectives
Identify linear functions and linear equations.

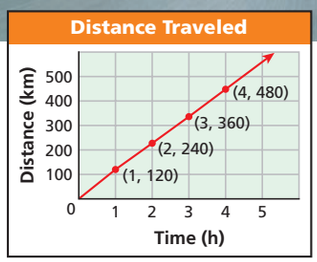
Graph linear functions that represent real-world situations and give their domain and range.

Vocabulary
linear function
linear equation

Why learn this?
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.



Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers y that the car would travel in x hours. Solutions are shown in the graph.

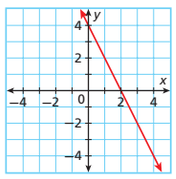


The graph represents a function because each domain value (x-value) is paired with exactly one range value (y-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.

EXAMPLE 1 **Identifying a Linear Function by Its Graph**

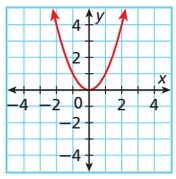
Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

A



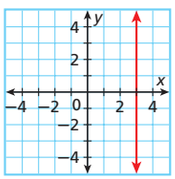
Each domain value is paired with exactly one range value. The graph forms a line.
linear function

B



Each domain value is paired with exactly one range value. The graph is not a line.
not a linear function

C



The only domain value, 3, is paired with many different range values.
not a function

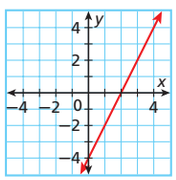
1a. Yes; each domain value is paired with exactly one range value; yes.

1b. Yes; each domain value is paired with exactly one range value; yes.

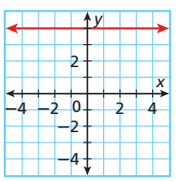
1c. No; each domain value is not paired with exactly one range value.

CHECK IT OUT! **Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?**

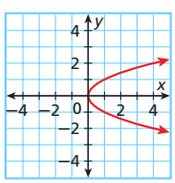
1a.



1b.



1c.



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1 Introduce

EXPLORATION

Identifying Linear Functions

The table shows the fees that a DVD rental company charges for movie rentals.

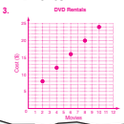
Movies	2	4	6	8	10
Cost ($)	8	12	16	20	24

1. Which is the independent variable and which is the dependent variable? ind.: number of movies; dep.: cost
2. For every increase of two movies, by how much does the cost increase? $4
3. Make a graph of the data.
4. What do you notice about the points that you plotted? They form a line.

THINK AND DISCUSS

5. **Explain** how you know this relationship is a function.
6. **Describe** how to use your graph to predict the cost of renting 12 movies.

3.



Motivate

Pose this situation: During a storm, snow fell at a rate of 0.5 in./h. The equation $y = 0.5x$ gives the number of inches of snow y that fell in x hours. Then have students graph ordered-pair solutions for this equation. Ask the following questions:

- What pattern do the points form? a line
- Is the relation a function? yes
- Why? Possible answer: Each x is paired with exactly one y.

Explorations and answers are provided in your online resources.

You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in x corresponds to a constant change in y.

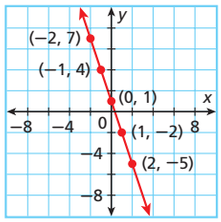
Caution!

If you find a constant change in the y-values, check for a constant change in the x-values. Both need to be constant for the function to be linear.

x	y	
−2	7	
−1	4	−3
0	1	−3
1	−2	−3
2	−5	−3
		+1 +1 +1 +1

In this table, a constant change of +1 in x corresponds to a constant change of −3 in y. These points satisfy a linear function.

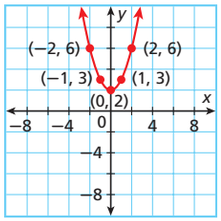
The points from this table lie on a line.



x	y	
−2	6	
−1	3	−3
0	2	−1
1	3	+1
2	6	+3
		+1 +1 +1 +1

In this table, a constant change of +1 in x does *not* correspond to a constant change in y. These points do *not* satisfy a linear function.

The points from this table do not lie on a line.



EXAMPLE 2 Identifying a Linear Function by Using Ordered Pairs

Tell whether each set of ordered pairs satisfies a linear function. Explain.

A $\{(2, 4), (5, 3), (8, 2), (11, 1)\}$

x	y	
2	4	
5	3	−1
8	2	−1
11	1	−1
		+3 +3 +3

Write the ordered pairs in a table. Look for a pattern.

A constant change of +3 in x corresponds to a constant change of −1 in y.

These points satisfy a linear function.

B $\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}$

x	y	
−10	10	
−5	4	−6
0	2	−2
5	0	−2
		+5 +5 +5

Write the ordered pairs in a table. Look for a pattern.

A constant change of +5 in x corresponds to different changes in y.

These points do not satisfy a linear function.

CHECK IT OUT! **2.** Tell whether the set of ordered pairs $\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}$ satisfies a linear function. Explain.

2. Yes; a constant change of +2 in x corresponds to a constant change of −1 in y.

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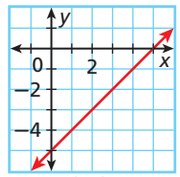
Additional Examples

Example 1

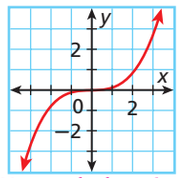
Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

A.



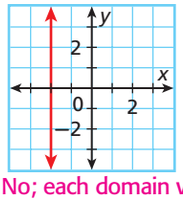
Yes; each domain value is paired with exactly one range value; yes.

B.



Yes; each domain value is paired with exactly one range value; no.

C.



No; each domain value is not paired with exactly one range value.

Example 2

Tell whether each set of ordered pairs satisfies a linear function. Explain.

A. $\{(0, -3), (4, 0), (8, 3), (12, 6), (16, 9)\}$ Yes; a constant change of +4 in x corresponds to a constant change of +3 in y.

B. $\{(-4, 13), (-2, 1), (0, -3), (2, 1), (4, 13)\}$ No; a constant change of +2 in x corresponds to different changes in y.

Also available online

2 Teach

Guided Instruction

Before working through **Examples 1** and **2,** review how to determine whether a relation is a function from ordered pairs and from a graph. In **Example 1B,** point out that the relation is a function, but not a linear function, because the graph is not a straight line. Introduce the standard form of a linear equation and practice writing several equations in standard form.

Teaching Tip **Reading Math** Point out that the word *linear* includes the word *line*.

ENGLISH LANGUAGE LEARNERS

Reaching All Learners

Through Visual Cues

Have students write five ordered pairs where the change between the x-values is constant. Have half the class select their y-values so that the change between each is also constant. Have the other half choose their y-values so that the change is not constant. Graph some examples of each on the board. Show that the points lie on a straight line only when the change is also constant between the y-values.

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- When is the graph of a line a function? When is it *not* a function?

EXAMPLE 2

- What is the relationship between change in x and change in y?

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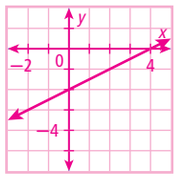
Additional Examples

Example 3

Tell whether each function is linear. If so, graph the function.

A. $x = 2y + 4$ yes



B. $xy = 4$ no

Also available online

Questioning Strategies

EXAMPLE 3

- What is the standard form of a linear equation?
- If an equation is in standard form, what are the exponents of x and y?

Technology Have students use a graphing calculator to graph the functions in **Example 3** to quickly verify which are linear and which are not.

Multiple Representations Students should be aware that there is more than one way (in fact, infinitely many ways) to write a linear equation in standard form. This is because for any linear equation $Ax + By = C$, you can multiply or divide both sides of the equation by any nonzero constant and the resulting equation will be equivalent. For instance, in **Example 3A**, you can multiply both sides of $-x + y = 3$ by -1 and get the equivalent equation $x - y = -3$, which is also in standard form.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation*. A **linear equation** is any equation that can be written in the *standard form* shown below.

Know it! Note

Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0

Notice that when a linear equation is written in standard form

- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

Linear		Not Linear	
$3x + 2y = 10$	*Standard form*	$3xy + x = 1$	*x and y are multiplied.*
$y - 2 = 3x$	*Can be written as $3x - y = -2$*	$x^3 + y = -1$	*x has an exponent other than 1.*
$-y = 5x$	*Can be written as $5x + y = 0$*	$x + \frac{6}{y} = 12$	*y is in a denominator.*

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

EXAMPLE 3 Graphing Linear Functions

Tell whether each function is linear. If so, graph the function.

A $y = x + 3$

$y = x + 3$ *Write the equation in standard form.*

$-x \quad -x$ *Subtraction Property of Equality*

$y - x = 3$

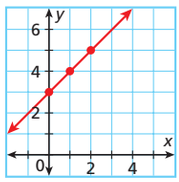
$-x + y = 3$ *The equation is in standard form ($A = -1$, $B = 1$, $C = 3$).*

The equation can be written in standard form, so the function is linear.

To graph, choose three values of x, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

x	$y = x + 3$	(x, y)
0	$y = 0 + 3 = 3$	$(0, 3)$
1	$y = 1 + 3 = 4$	$(1, 4)$
2	$y = 2 + 3 = 5$	$(2, 5)$

Plot the points and connect them with a straight line.



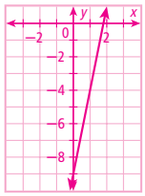
B $y = x^2$

This is not linear, because x has an exponent other than 1.

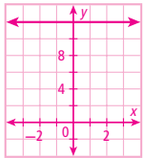
Remember!

- $y - x = y + (-x)$
- $y + (-x) = -x + y$
- $-x = -1x$
- $y = 1y$

3a. yes



3b. yes



CHECK IT OUT!

Tell whether each function is linear. If so, graph the function.

3a. $y = 5x - 9$ **3b.** $y = 12$ **3c.** $y = 2^x$ no

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

EXAMPLE 4 *Career Application*

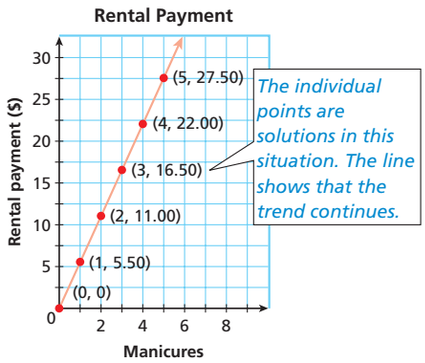
Sue rents a manicure station in a salon and pays the salon owner $5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where x is the number of manicures. Graph this function and give its domain and range.

Choose several values of x and make a table of ordered pairs.

x	$f(x) = 5.50x$
0	$f(0) = 5.50(0) = 0$
1	$f(1) = 5.50(1) = 5.50$
2	$f(2) = 5.50(2) = 11.00$
3	$f(3) = 5.50(3) = 16.50$
4	$f(4) = 5.50(4) = 22.00$
5	$f(5) = 5.50(5) = 27.50$

The number of manicures must be a whole number, so the domain is $\{0, 1, 2, 3, \ldots\}$. The range is $\{0, 5.50, 11.00, 16.50, \ldots\}$.

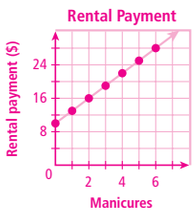
Graph the ordered pairs.



Remember!

$f(x) = y$, so in Example 4, graph the function values (dependent variable) on the y-axis.

4.



D: {0, 1, 2, 3, …}

R: {$10, $13, $16, $19, …}

CHECK IT OUT!

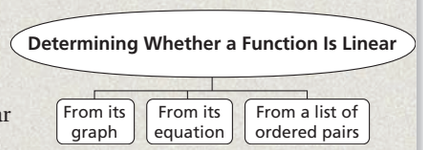
4. What if…? At another salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function and give its domain and range.

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?
2. In Example 4, why is every point on the line not a solution?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.

Know it! Note



COMMON ERROR ALERT

Students might think that any equation with x and y on the left side is in standard form. Remind them of the other conditions: the exponents of x and y are 1; x and y are not multiplied together; and x and y are not in denominators, exponents, or radical signs.

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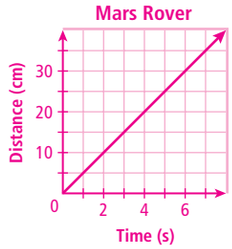
Additional Examples

Example 4

The distance in centimeters that a Mars rover can travel at top speed is given by the function $f(x) = 5x$, where x is the time in seconds. Graph this function and give its domain and range.



D: $x \geq 0$; R: $y \geq 0$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- How do you decide what a reasonable domain is?

3 Close

Summarize

Remind students that a function may or may not be linear, and that a linear equation may or may not represent a function.

For example:

- $y = x + 2$ is a linear function.
- $y = x^2$ is a function but is not linear.
- $x = 4$ is a linear equation but is not a function.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. No; all the points of the function must form a line in order for it to be a linear function.
2. It is only possible to do a whole number of manicures, so the points whose x-coordinates are not whole numbers have no meaning in this situation.
3. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
- **Basic** 15–20, 26–29
- **Average** 15–20, 26–29, 60
- **Advanced** 15–20, 26–29, 60

If you finished Examples **1–4**
- **Basic** 15–49, 54, 55, 57–59
- **Average** 15–25, 30–54, 53–63
- **Advanced** 15–25, 30–54, 53–63

Homework Quick Check
Quickly check key concepts.
Exercises: 16, 18, 24, 25, 34, 36, 38

Answers

2. Each domain value is paired with exactly one range value.
3. Each domain value is paired with exactly one range value.
4. Each domain value is paired with exactly one range value.
5. A constant change of -1 in x corresponds to a constant change of $+2$ in y.
6. There is a constant change in y, but there is not a corresponding constant change in x.
7. A constant change of -2 in x corresponds to a constant change of -2 in y.
8. For explanation, see Additional Answers.
9, 10, 12. For graphs, see Additional Answers.
13, 14. See Additional Answers.
15–20. For explanations, see Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 13–14, 25, 50–51, 54, 55, 61–63

Construct viable arguments and critique the reasoning of others. Exercises 52–53, 56, 59

Look for and make use of structure. Exercises 2–12, 15–24, 26–49, 60

Learn It Online
State Resources Online

4-1 Exercises

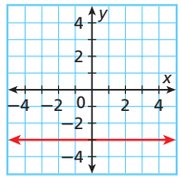
Learn It Online
Homework Help Online
Parent Resources Online

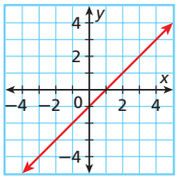
GUIDED PRACTICE

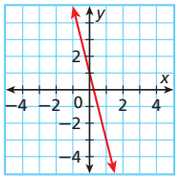
1. **Vocabulary** Is the *linear equation* $3x - 2 = y$ in standard form? Explain.
No; it is not in the form $Ax + By = C$.

SEE EXAMPLE 1

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

2.  yes; yes

3.  yes; yes

4.  yes; yes

SEE EXAMPLE 2

Tell whether the given ordered pairs satisfy a linear function. Explain.

5. yes

x	5	4	3	2	1
y	0	2	4	6	8

6. no

x	1	4	9	16	25
y	1	2	3	4	5

7. $\{(0, 5), (-2, 3), (-4, 1), (-6, -1), (-8, -3)\}$ yes
8. $\{(2, -2), (-1, 0), (-4, 1), (-7, 3), (-10, 6)\}$ no

SEE EXAMPLE 3

Tell whether each function is linear. If so, graph the function.

9. $2x + 3y = 5$ yes
10. $2y = 8$ yes
11. $\frac{x^2 + 3}{5} = y$ no
12. $\frac{x}{5} = \frac{y}{3}$ yes

SEE EXAMPLE 4

13. **Transportation** A train travels at a constant speed of 75 mi/h. The function $f(x) = 75x$ gives the distance that the train travels in x hours. Graph this function and give its domain and range.

14. **Entertainment** A movie rental store charges a \$6.00 membership fee plus \$2.50 for each movie rented. The function $f(x) = 2.50x + 6$ gives the cost of renting x movies. Graph this function and give its domain and range.

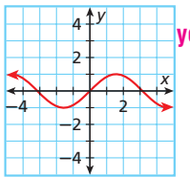
PRACTICE AND PROBLEM SOLVING

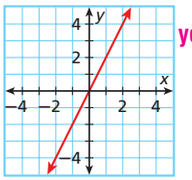
Independent Practice

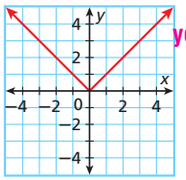
For Exercises	See Example
15–17	1
18–20	2
21–24	3
25	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

15.  yes; no

16.  yes; yes

17.  yes; no

Tell whether the given ordered pairs satisfy a linear function. Explain.

18. no

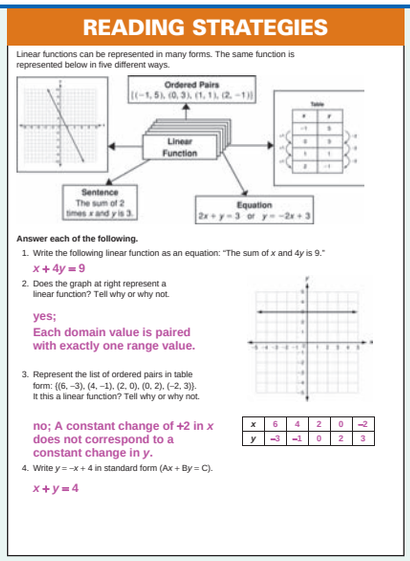
x	−3	0	3	6	9
y	−2	−1	0	2	4

19. yes

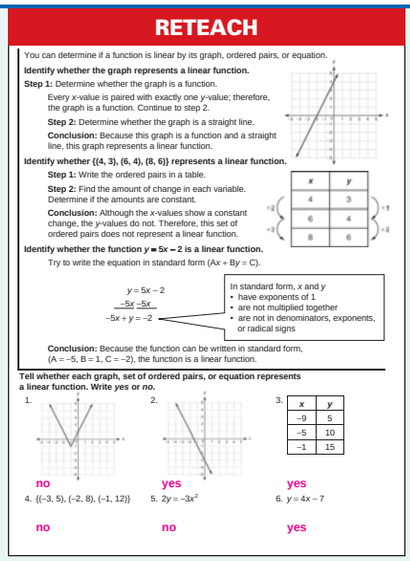
x	−1	0	1	2	3
y	−3	−2	−1	0	1

20. $\{(3, 4), (0, 2), (-3, 0), (-6, -2), (-9, -4)\}$ yes

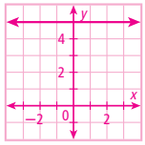
READING STRATEGIES



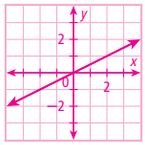
RETEACH



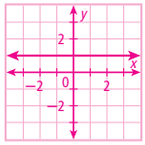
21. yes



22. yes



24. yes



27. yes; yes

Tell whether each function is linear. If so, graph the function.

21. $y = 5$ **22.** $4y - 2x = 0$ **23.** $\frac{3}{x} + 4y = 10$ **no** **24.** $5 + 3y = 8$

25. Transportation The gas tank in Tony's car holds 15 gallons, and the car can travel 25 miles for each gallon of gas. When Tony begins with a full tank of gas, the function $f(x) = -\frac{1}{25}x + 15$ gives the amount of gas $f(x)$ that will be left in the tank after traveling x miles (if he does not buy more gas). Graph this function and give its domain and range.

Tell whether the given ordered pairs satisfy a function. If so, is it a linear function?

26. $\{(2, 5), (2, 4), (2, 3), (2, 2), (2, 1)\}$ **no; no** **27.** $\{(-8, 2), (-6, 0), (-4, -2), (-2, -4), (0, -6)\}$

28.

x	−10	−6	−2	2	4
y	0	0.25	0.50	0.75	1

yes; no

29.

x	−5	−1	3	7	11
y	1	1	1	1	1

yes; yes

Tell whether each equation is linear. If so, write the equation in standard form and give the values of *A*, *B*, and *C*.

30. $2x - 8y = 16$ **31.** $y = 4x + 2$ **32.** $2x = \frac{y}{3} - 4$ **33.** $\frac{4}{x} = y$ **no**

34. $\frac{x+4}{2} = \frac{y-4}{3}$ **35.** $x = 7$ **36.** $xy = 6$ **no** **37.** $3x - 5 + y = 2y - 4$

38. $y = -x + 2$ **39.** $5x = 2y - 3$ **40.** $2y = -6$ **41.** $y = \sqrt{x}$ **no**

Graph each linear function.

42. $y = 3x + 7$ **43.** $y = x + 25$ **44.** $y = 8 - x$ **45.** $y = 2x$

46. $-2y = -3x + 6$ **47.** $y - x = 4$ **48.** $y - 2x = -3$ **49.** $x = 5 + y$

50. Measurement One inch is equal to approximately 2.5 centimeters. Let x represent inches and y represent centimeters. Write an equation in standard form relating x and y. Give the values of A, B, and C. **$2.5x - y = 0$; $A = 2.5$, $B = -1$, $C = 0$**

51. Wages Molly earns $8.00 an hour at her job.

a. Let x represent the number of hours that Molly works. Write a function using x and $f(x)$ that describes Molly's pay for working x hours. **$f(x) = 8x$**

b. Graph this function and give its domain and range.

52. Write About It For $y = 2x - 1$, make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.

53. Critical Thinking Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.

MULTI-STEP TEST PREP

Victoria Smith/HMH

54. a. Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.

b. Create a graph of the data.

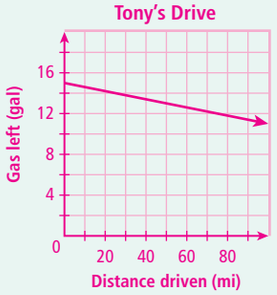
c. How can you tell from the graph that the relationship is linear? **The graph forms a line.**

Time (min)	Calories
3	27
6	54
9	81
12	108
15	135
18	162
21	189

MULTI-STEP TEST PREP **Exercise 54** involves creating a graph from a table and determining whether the relationship is linear. This exercise prepares students for the Multi-Step Test Prep.

Answers

25.



D: $0 \le x \le 375$; R: $0 \le y \le 15$

30. yes; $2x - 8y = 16$; $A = 2$; $B = -8$; $C = 16$

31. yes; $-4x + y = 2$; $A = -4$; $B = 1$; $C = 2$

32. yes; $2x - \frac{1}{3}y = -4$; $A = 2$; $B = -\frac{1}{3}$; $C = -4$

34. yes; $3x - 2y = -20$; $A = 3$; $B = -2$; $C = -20$

35. yes; $x = 7$; $A = 1$; $B = 0$; $C = 7$

37. yes; $3x - y = 1$; $A = 3$; $B = -1$; $C = 1$

38. yes; $x + y = 2$; $A = 1$, $B = 1$, $C = 2$

39. yes; $5x - 2y = -3$; $A = 5$, $B = -2$, $C = -3$

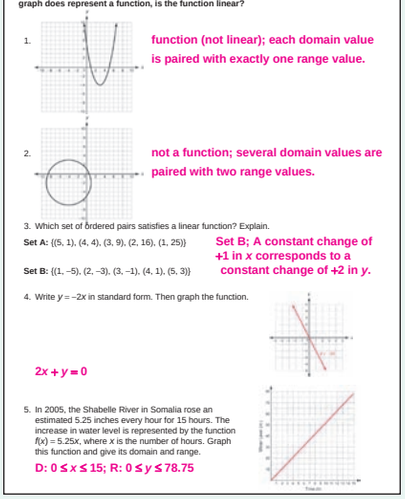
40. yes; $2y = -6$; $A = 0$, $B = 2$, $C = -6$

42–49, 51b–54b. See Additional Answers.

4-1 PRACTICE A

4-1 PRACTICE C

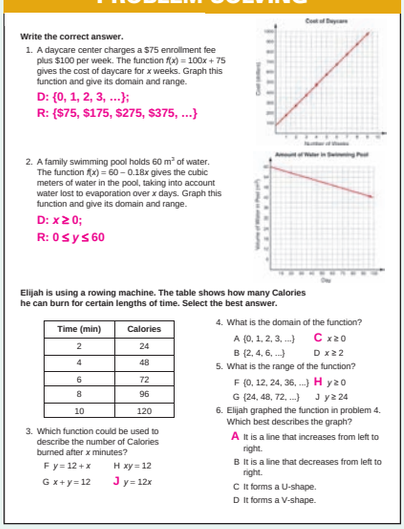
4-1 PRACTICE B



Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

1. function (not linear); each domain value is paired with exactly one range value.
2. not a function; several domain values are paired with two range values.
3. Which set of ordered pairs satisfies a linear function? Explain.
   Set A: {(5, 1), (4, 4), (3, 9), (2, 16), (1, 25)}
   Set B: {(1, −5), (2, −3), (3, −1), (4, 1), (5, 3)}
   Set B; A constant change of +1 in x corresponds to a constant change of +2 in y.
4. Write $y = -2x$ in standard form. Then graph the function.
   $2x + y = 0$
5. In 2005, the Shabelle River in Somalia rose an estimated 5.25 inches every hour for 15 hours. The increase in water level is represented by the function $f(x) = 5.25x$, where x is the number of hours. Graph this function and give its domain and range.
   D: $0 \le x \le 15$; R: $0 \le y \le 78.75$

PROBLEM SOLVING



Write the correct answer.

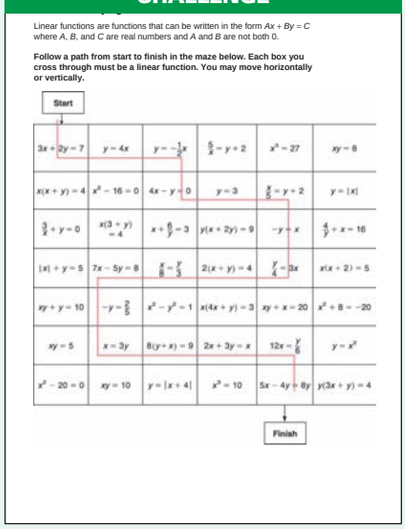
1. A daycare center charges a $75 enrollment fee plus $100 per week. The function $f(x) = 100x + 75$ gives the cost of daycare for x weeks. Graph this function and give its domain and range.
   D: {0, 1, 2, 3, ...};
   R: {$75, $175, $275, $375, ...}
2. A family swimming pool holds 60 m^3 of water. The function $f(x) = 60 - 0.18x$ gives the cubic meters of water in the pool, taking into account water lost to evaporation over x days. Graph this function and give its domain and range.
   D: $x \ge 0$;
   R: $0 \le y \le 60$

Elijah is using a rowing machine. The table shows how many Calories he can burn for certain lengths of time. Select the best answer.

Time (min)	Calories
2	24
4	48
6	72
8	96
10	120

3. Which function could be used to describe the number of Calories burned after x minutes?
   F $y = 12 + x$ H $xy = 12$
   G $x + y = 12$ J $y = 12x$
4. What is the domain of the function?
   A {0, 1, 2, 3, ...} C $x \ge 0$
   B {2, 4, 6, ...} D $x \ge 2$
5. What is the range of the function?
   F {0, 12, 24, 36, ...} H $y \ge 0$
   G {24, 48, 72, ...} J $y \ge 24$
6. Elijah graphed the function in problem 4. Which best describes the graph?
   A It is a line that increases from left to right.
   B It is a line that decreases from left to right.
   C It forms a U-shape.
   D It forms a V-shape.

CHALLENGE



Linear functions are functions that can be written in the form $Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0.

Follow a path from start to finish in the maze below. Each box you cross through must be a linear function. You may move horizontally or vertically.

Start

$3x + 2y = 7$	$y = 4x$	$y = -\frac{1}{2}x$	$\frac{x}{5} - y + 2$	$x^3 = 27$	$xy = 8$
$x(x + y) = 4$	$x^2 - 16 = 0$	$4x - y = 0$	$y = 3$	$\frac{x}{5} = y + 2$	$y = \lvert x \rvert$
$\frac{3}{x} + y = 0$	$\frac{x(3+y)}{-4}$	$x + \frac{6}{5} = 3$	$y(x + 2y) = 9$	$-y = x$	$\frac{4}{y} + x = 16$
$\lvert x \rvert + y = 5$	$7x - 5y = 8$	$\frac{x}{6} = \frac{y}{5}$	$2(x + y) = 4$	$\frac{y}{4} = 3x$	$x(x + 2) = 5$
$xy + y = 10$	$-y = \frac{2}{5}$	$x^2 - y^2 = 1$	$x(4x + y) = 3$	$xy + x = 20$	$x^2 + 8 = -20$
$xy = 5$	$x = 3y$	$8(y + x) = 9$	$2x + 3y = x$	$12x = \frac{y}{6}$	$y = x^2$
$x^2 - 20 = 0$	$xy = 10$	$y = \lvert x + 4 \rvert$	$x^3 = 10$	$5x - 4y = 8y$	$y(3x + y) = 4$

Finish

TEST PREP DOCTOR Students who did not choose **C** in **Exercise 57** may not understand that in a linear function, x or y cannot be in the denominator of a fraction.

Journal

Have students explain how to identify a linear function from its graph and from its equation.

ALTERNATIVE ASSESSMENT

Have students write a real-world problem that can be represented by a linear function. Then have them write the function rule and determine a reasonable domain and range.

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4-1 Lesson Quiz

Tell whether each set of ordered pairs satisfies a linear function. Explain.

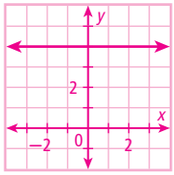
1. $\{(-3, 10), (-1, 9), (1, 7), (3, 4), (5, 0)\}$ No; a constant change of +2 in x corresponds to different changes in y.

2. $\{(3, 4), (5, 7), (7, 10), (9, 13), (11, 16)\}$ Yes; a constant change of +2 in x corresponds to a constant change of +3 in y.

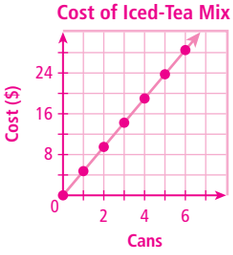
Tell whether each function is linear. If so, graph the function.

3. $y = 3 - 2^x$ no

4. $3y = 12$ yes



5. The cost of a can of iced-tea mix at SaveMore Grocery is \$4.75. The function $f(x) = 4.75x$ gives the cost of x cans of iced-tea mix. Graph this function and give its domain and range.



D: {0, 1, 2, 3, . . .}
R: {0, 4.75, 9.50, 14.25, . . .}

Also available online

55. Physical Science A ball was dropped from a height of 100 meters. Its height above the ground in meters at different times after its release is given in the table. Do these ordered pairs satisfy a linear function? Explain.

Time (s)	0	1	2	3
Height (m)	100	90.2	60.8	11.8

No; each constant change in time (+ 1 s) is not accompanied by a constant change in height.

56. Critical Thinking Is the equation $x = 9$ a linear equation? Does it describe a linear function? Explain. Yes; the equation can be written in stand. form with $A = 1$, $B = 0$, and $C = 9$. No; all solutions are ordered pairs with x-value 9. The x-value 9 corresponds to more than one y-value.

TEST PREP

57. Which is NOT a linear function?

Ⓐ $y = 8x$ Ⓑ $y = x + 8$ Ⓒ $y = \frac{8}{x}$ Ⓓ $y = 8 - x$

58. The speed of sound in 0 °C air is about 331 feet per second. Which function could be used to describe the distance in feet d that sound will travel in air in s seconds?

Ⓕ $d = s + 331$ Ⓖ $d = 331s$ Ⓗ $s = 331d$ Ⓙ $s = 331 - d$

59. Extended Response Write your own linear function. Show that it is a linear function in at least three different ways. Explain any connections you see between your three methods.

CHALLENGE AND EXTEND

60. What equation describes the x-axis? the y-axis? Do these equations represent linear functions?

$y = 0$; $x = 0$; the first describes a linear function, but the second does not.

Geometry Copy and complete each table below. Then tell whether the table shows a linear relationship.

61.

Perimeter of a Square	
Side Length	Perimeter
1	4
2	8
3	12
4	16

linear

62.

Area of a Square	
Side Length	Area
1	1
2	4
3	9
4	16

not linear

63.

Volume of a Cube	
Side Length	Volume
1	1
2	8
3	27
4	64

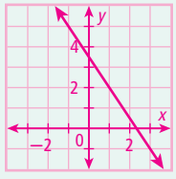
not linear

Answers

59. Possible answer: $3x + 2y = 7$; it is a linear equation because it can be written in standard form with $A = 3$, $B = 2$, and $C = 7$.
A table of values also shows it is a linear function:

x	y
−3	8
−2	6.5
−1	5
0	3.5
1	2
2	0.5

Each constant change in x (+1) is accompanied by a constant change in y (−1.5).
The graph shows a linear function.



Both the graph and the table show solutions to the equation.

COMMON CORE

4-2 Using Intercepts

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases.…*
a. Graph linear … functions and show intercepts, …. *Also* **CC.9-12.A.CED.2*, CC.9-12.A.CED.3*, CC.9-12.F.IF.2, CC.9-12.F.IF.4*, CC.9-12.F.IF.5***

Objectives
Find *x*- and *y*-intercepts and interpret their meanings in real-world situations.

Use *x*- and *y*-intercepts to graph lines.

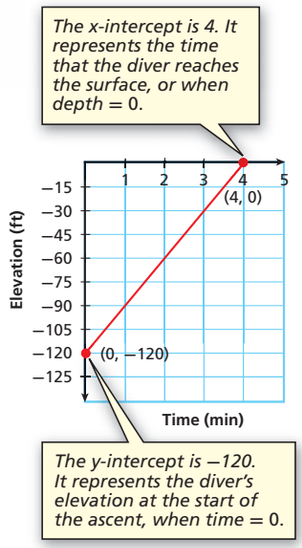
Vocabulary
y-intercept
x-intercept

Who uses this?
Divers can use intercepts to determine the time a safe ascent will take.

A diver explored the ocean floor 120 feet below the surface and then ascended at a rate of 30 feet per minute. The graph shows the diver's elevation below sea level during the ascent.

The ***y*-intercept** is the *y*-coordinate of the point where the graph intersects the *y*-axis. The *x*-coordinate of this point is always 0.

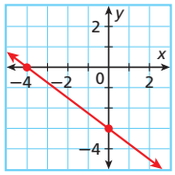
The ***x*-intercept** is the *x*-coordinate of the point where the graph intersects the *x*-axis. The *y*-coordinate of this point is always 0.



EXAMPLE 1 Finding Intercepts

Find the *x*- and *y*-intercepts.

A



The graph intersects the x-axis at (−4, 0).
The *x*-intercept is −4.

The graph intersects the y-axis at (0, −3).
The *y*-intercept is −3.

B $3x - 2y = 12$

To find the *x*-intercept, replace *y* with 0 and solve for *x*.

$3x - 2y = 12$
$3x - 2(0) = 12$
$3x - 0 = 12$
$3x = 12$
$\frac{3x}{3} = \frac{12}{3}$
$x = 4$

The *x*-intercept is 4.

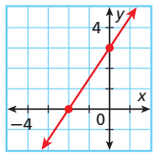
To find the *y*-intercept, replace *x* with 0 and solve for *y*.

$3x - 2y = 12$
$3(0) - 2y = 12$
$0 - 2y = 12$
$-2y = 12$
$\frac{-2y}{-2} = \frac{12}{-2}$
$y = -6$

The *y*-intercept is −6.

CHECK IT OUT! **Find the *x*- and *y*-intercepts.**

1a. *x*-int.: −2; *y*-int.: 3



1b. $-3x + 5y = 30$ *x*-int.: −10; *y*-int.: 6

1c. $4x + 2y = 16$ *x*-int.: 4; *y*-int.: 8

4-2 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Find *x*- and *y*-intercepts and interpret their meanings in real-world situations.

Use *x*- and *y*-intercepts to graph lines.

Online Edition
Tutorial Videos

Countdown

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Warm Up

Solve each equation.

1. $5x + 0 = -10$ −2
2. $33 = 0 + 3y$ 11
3. $\frac{4}{5}x + \frac{1}{10} = \frac{9}{10}$ 1
4. $2x + 14 = -3x + 4$ −2
5. $-5y - 1 = 7y + 5$ $-\frac{1}{2}$

Also available online

Math Humor

Teacher: Where's the graph of your function?

Student: It was intercepted on the way to school.

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *Also* **CC.9-12.A.CED.2*, CC.9-12.A.CED.3*, CC.9-12.F.IF.2, CC.9-12.F.IF.4*, CC.9-12.F.IF.5***

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Using Intercepts

Katarina's family is traveling from Brownsville, Texas, to their home in Laredo, Texas. The distance from Brownsville to Laredo is 200 miles, and Katarina's mother is driving at a speed of 50 miles per hour.

1. Complete the table.

Time from Start of Trip (h)	0	1	2	3	4
Distance to Laredo (mi)	200	150	100	50	0

2. Make a graph of the data.
3. What is the starting distance? What point on the graph corresponds to the starting distance? Where is this point located? 200 mi; (0, 200); *y*-axis
4. How long does it take to reach Laredo? What point on the graph corresponds to this time? Where is this point located? 4 h; (4, 0); *x*-axis

THINK AND DISCUSS

5. **Explain** how you could use a graph to find the time it would take to reach Laredo if Katarina's mother drove at a speed of 40 miles per hour.
6. **Describe** what it means for a point to lie on the *y*-axis in a graph of distance versus time.

2.

Motivate

Have students graph $3x + 4y = 12$ by generating ordered pairs for *x* and *y* using the following table.

x	*y*
−2	4.5
0	3
2	1.5
4	0

Then have students circle the rows of the table that include 0. Ask students where these points are located on the graph. the axes

Explorations and answers are provided in your online resources.

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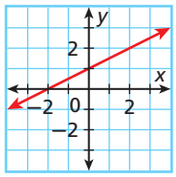
Additional Examples

Example 1

Find the *x*- and *y*-intercepts.

A.



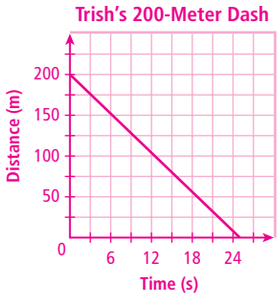
x-int.: −2; *y*-int.: 1

B. $5x - 2y = 10$

x-int.: 2; *y*-int.: −5

Example 2

Trish can run the 200 m dash in 25 s. The function $f(x) = 200 - 8x$ gives the distance remaining to run after x seconds. Graph this function and find the intercepts. What does each intercept represent?



x-int.: 25; *y*-int.: 200

x-int.: the time when Trish finishes the race, when the distance remaining is 0

y-int.: the number of meters Trish has to run at the start of the race, when the time passed is 0

Also available online

Student to Student

Finding Intercepts

Madison Stewart
Jefferson High School

I use the "cover-up" method to find intercepts. To use this method, make sure the equation is in standard form first.

If I have $4x - 3y = 12$:

First, I cover $4x$ with my finger and solve the equation I can still see.

$\square - 3y = 12$
$y = -4$

The y-intercept is −4.

Then I cover $-3y$ with my finger and do the same thing.

$4x - \square = 12$
$x = 3$

The x-intercept is 3.

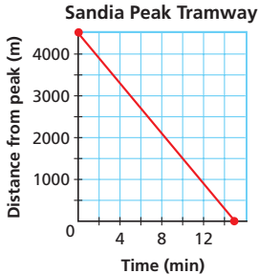
EXAMPLE 2 Travel Application

The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function $f(x) = 4500 - 300x$ gives the tram's distance in meters from the top of the peak after x minutes. Graph this function and find the intercepts. What does each intercept represent?

Neither time nor distance can be negative, so choose several nonnegative values for x. Use the function to generate ordered pairs.

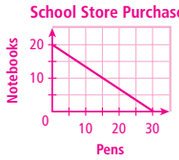
x	0	2	5	10	15
$f(x) = 4500 - 300x$	4500	3900	3000	1500	0

Graph the ordered pairs. Connect the points with a line.



- *y*-intercept: 4500. This is the starting distance from the top (time = 0).
- *x*-intercept: 15. This the time when the tram reaches the peak (distance = 0).

2a.



x-int.: 30; *y*-int.: 20

2b. ***x*-int.: pens that can be purchased if no notebooks are purchased; *y*-int.: notebooks that can be purchased if no pens are purchased**

Caution!

The graph is not the path of the tram. Even though the line is descending, the graph describes the distance from the peak as the tram goes *up* the mountain.

CHECK IT OUT!

2. The school store sells pens for $2.00 and notebooks for $3.00. The equation $2x + 3y = 60$ describes the number of pens x and notebooks y that you can buy for $60.

a. Graph the function and find its intercepts.

b. What does each intercept represent?

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Mathematical Practices Questioning Strategies

EXAMPLE 1

- What point on a graph corresponds to the *x*-intercept? the *y*-intercept?
- What intercept do you find by substituting zero for *x*?

EXAMPLE 2

- Why is the graph of only the first quadrant shown?
- What do other points on the graph, such as (5, 160), (10, 120), and so on, represent?

2 Teach

Guided Instruction

Show how to find the intercepts from a graph and then from an equation. Show students the "cover-up" method of finding intercepts from an equation (described in Student to Student) and ask students why it works. The value covered up always becomes zero.

Teaching Tip **Reading Math** English Language Learners

Have students familiar with football explain the meaning of *interception* in that sport. Then ask the class how it relates to an intercept in math.

Reaching All Learners

Through Number Sense

Give students the following scenario: *You have $25, and you spend $5 per week.* Tell students that the function $y = 25 - 5x$ describes the amount of money y they will have after x weeks. Have them find ordered pairs for $x = 0, 1, 2, 3, 4$, and 5. (0, 25), (1, 20), (2, 15), (3, 10), (4, 5), (5, 0) Graph the ordered pairs, labeling each as follows: (week 1, $20 left).

Discuss why the value of y when $x = 6$ is not reasonable in this situation. You have no money left to spend after the 5th week.

Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

EXAMPLE 3 Graphing Linear Equations by Using Intercepts

Use intercepts to graph the line described by each equation.

A $2x - 4y = 8$

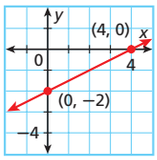
Step 1 Find the intercepts.

x-intercept:	*y*-intercept:
$2x - 4y = 8$	$2x - 4y = 8$
$2x - 4(0) = 8$	$2(0) - 4y = 8$
$2x = 8$	$-4y = 8$
$\frac{2x}{2} = \frac{8}{2}$	$\frac{-4y}{-4} = \frac{8}{-4}$
$x = 4$	$y = -2$

Step 2 Graph the line.

Plot (4, 0) and (0, −2).

Connect with a straight line.



B $\frac{2}{3}y = 4 - \frac{1}{2}x$

Step 1 Write the equation in standard form.

$6\left(\frac{2}{3}y\right) = 6\left(4 - \frac{1}{2}x\right)$ *Multiply both sides by 6, the LCD of the fractions, to clear the fractions.*

$4y = 24 - 3x$

$3x + 4y = 24$ *Write the equation in standard form.*

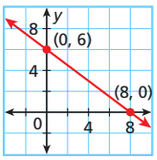
Step 2 Find the intercepts.

x-intercept:	*y*-intercept:
$3x + 4y = 24$	$3x + 4y = 24$
$3x + 4(0) = 24$	$3(0) + 4y = 24$
$3x = 24$	$4y = 24$
$\frac{3x}{3} = \frac{24}{3}$	$\frac{4y}{4} = \frac{24}{4}$
$x = 8$	$y = 6$

Step 3 Graph the line.

Plot (8, 0) and (0, 6).

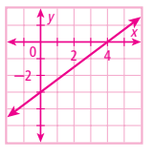
Connect with a straight line.



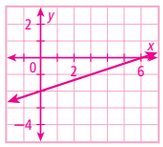
Helpful Hint

You can use a third point to check your line. Either choose a point from your graph and check it in the equation, or use the equation to generate a point and check that it is on your graph.

3a.



3b.



CHECK IT OUT! **Use intercepts to graph the line described by each equation.**

3a. $-3x + 4y = -12$

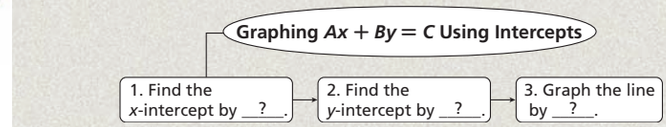
3b. $y = \frac{1}{3}x - 2$

MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. A function has *x*-intercept 4 and *y*-intercept 2. Name two points on the graph of this function.
2. What is the *y*-intercept of $2.304x + y = 4.318$? What is the *x*-intercept of $x - 92.4920y = -21.5489$?
3. **GET ORGANIZED** Copy and complete the graphic organizer.

Know it! Note



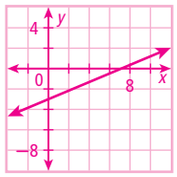
Power Presentations with PowerPoint®

Additional Examples

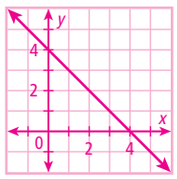
Example 3

Use intercepts to graph the line described by each equation.

A. $3x - 7y = 21$



B. $y = -x + 4$



Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- What is the difference between graphing a line using intercepts and graphing a line by generating a table of ordered pairs?

3 Close

Summarize

Review the definitions of *x*- and *y*-intercepts and how to find them from a graph and from an equation.

Tell students that finding intercepts is especially useful for graphing lines, because it is often quicker than generating ordered pairs, especially when the equation is written in standard form.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. (4, 0) and (0, 2)
2. 4.318; −21.5489
3. See Additional Answers.

4-2 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

Assign *Guided Practice* exercises as necessary.

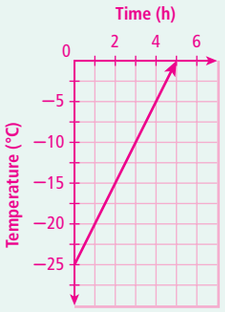
If you finished Examples **1–3**

Basic 13–29, 34–38, 40–42
Average 13–33, 38–42, 44, 46
Advanced 14–20 even, 22–33, 38–47

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 18, 22, 24, 26, 30

Answers

8a. Refrigeration Tank Temperature



b. *x*-int.: time when temp. = 0 °C; *y*-int.: initial temp.

9–12. See Additional Answers.

22a. For graph, see Additional Answers.

b. See Additional Answers.

23a. For graph, see Additional Answers.

b. See Additional Answers.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 8, 22–23, 30–32, 38

Construct viable arguments and critique the reasoning of others. Exercises 33, 39

Use appropriate tools strategically. Exercises 2–7, 9–12, 13–21, 24–29

Look for and make use of structure. Exercises 34–37, 46

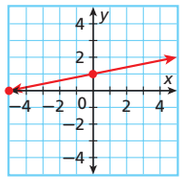
Learn It Online
State Resources Online

GUIDED PRACTICE

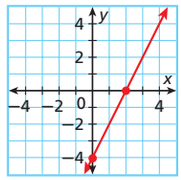
1. Vocabulary The ____?____ is the *y*-coordinate of the point where a graph crosses the *y*-axis. (*x-intercept* or *y-intercept*) ***y*-intercept**

SEE EXAMPLE 1

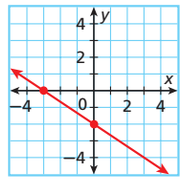
Find the *x*- and *y*-intercepts.

2. 

***x*-int.: −5; *y*-int.: 1**

3. 

***x*-int.: 2; *y*-int.: −4**

4. 

***x*-int.: −3; *y*-int.: −2**

5. $2x - 4y = 4$ ***x*-int.: 2; *y*-int.: −1**

6. $-2y = 3x - 6$ ***x*-int.: 2; *y*-int.: 3**

7. $4y + 5x = 2y - 3x + 16$ ***x*-int.: 2; *y*-int.: 8**

SEE EXAMPLE 2

8. Biology To thaw a specimen stored at −25 °C, the temperature of a refrigeration tank is raised 5 °C every hour. The temperature in the tank after x hours can be described by the function $f(x) = -25 + 5x$.

a. Graph the function and find its intercepts. ***x*-int.: 5; *y*-int.: −25**

b. What does each intercept represent?

SEE EXAMPLE 3

Use intercepts to graph the line described by each equation.

9. $4x - 5y = 20$ **10.** $y = 2x + 4$ **11.** $\frac{1}{3}x - \frac{1}{4}y = 2$ **12.** $-5y + 2x = -10$

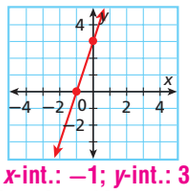
PRACTICE AND PROBLEM SOLVING

Independent Practice

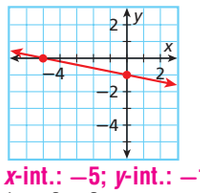
For Exercises	See Example
13–21	1
22–23	2
24–29	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

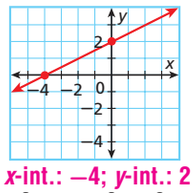
Find the *x*- and *y*-intercepts.

13. 

***x*-int.: −1; *y*-int.: 3**

14. 

***x*-int.: −5; *y*-int.: −1**

15. 

***x*-int.: −4; *y*-int.: 2**

16. $6x + 3y = 12$ **17.** $4y - 8 = 2x$ **18.** $-2y + x = 2y - 8$

19. $4x + y = 8$ **20.** $y - 3x = -15$ **21.** $2x + y = 10x - 1$

22. Environmental Science A fishing lake was stocked with 300 bass. Each year, the population decreases by 25. The population of bass in the lake after x years is represented by the function $f(x) = 300 - 25x$. ***x*-int.: 12; *y*-int.: 300**

a. Graph the function and find its intercepts.

b. What does each intercept represent?

23. Sports Julie is running a 5-kilometer race. She runs 1 kilometer every 5 minutes. Julie's distance from the finish line after x minutes is represented by the function $f(x) = 5 - \frac{1}{5}x$.

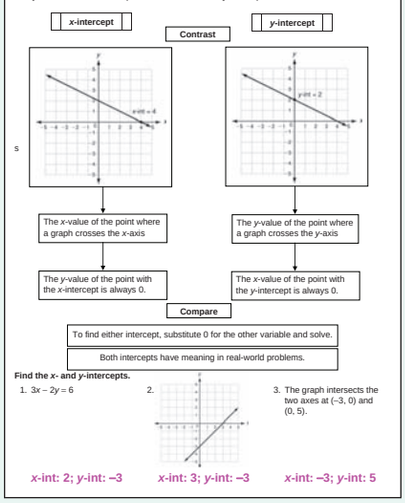
a. Graph the function and find its intercepts. ***x*-int.: 25; *y*-int.: 5**

b. What does each intercept represent?

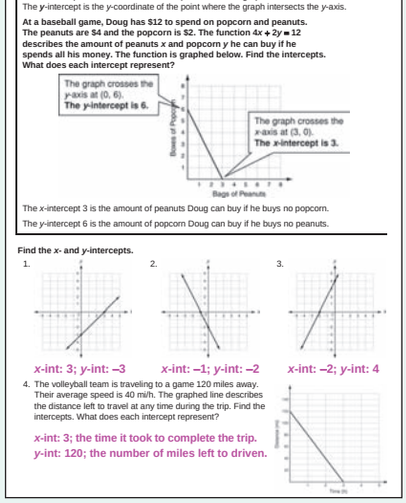
16. ***x*-int.: 2; *y*-int.: 4**
17. ***x*-int.: −4; *y*-int.: 2**
18. ***x*-int.: −8; *y*-int.: 2**
19. ***x*-int.: 2; *y*-int.: 8**
20. ***x*-int.: 5; *y*-int.: −15**
21. ***x*-int.: $\frac{1}{8}$; *y*-int.: −1**

READING STRATEGIES

Study the chart below to compare and contrast *x*- and *y*-intercepts.



RETEACH



Use intercepts to graph the line described by each equation.

24. $4x - 6y = 12$ **25.** $2x + 3y = 18$ **26.** $\frac{1}{2}x - 4y = 4$

27. $y - x = -1$ **28.** $5x + 3y = 15$ **29.** $x - 3y = -1$

Biology

Bamboo is the world's fastest-growing woody plant. Some varieties can grow more than 30 centimeters a day and up to 40 meters tall.

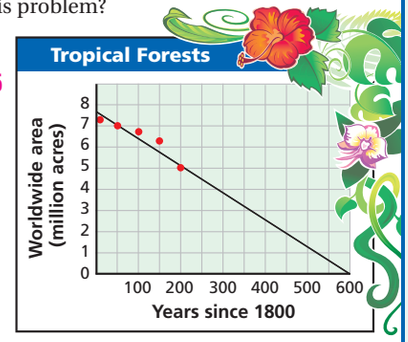
30. Biology A bamboo plant is growing 1 foot per day. When you first measure it, it is 4 feet tall.

a. Write an equation to describe the height y, in feet, of the bamboo plant x days after you measure it. $y = x + 4$

b. What is the y-intercept? 4

c. What is the meaning of the y-intercept in this problem? orig. height of bamboo plant

31. Estimation Look at the scatter plot and trend line. x-int.: 600; y-int.: 7.5



a. Estimate the x- and y-intercepts.

b. What is the real-world meaning of each intercept?

32. Personal Finance A bank employee notices an abandoned checking account with a balance of \$412. If the bank charges a \$4 monthly fee for the account, the function $b = 412 - 4m$ shows the balance b in the account after m months.

32b. x-int.: 103; y-int.: 412; y-int.: balance when bank employee noticed account; x-int.: number of months from that time until account has \$0

a. Graph the function and give its domain and range. (*Hint:* The bank will keep charging the monthly fee even after the account is empty.)

b. Find the intercepts. What does each intercept represent?

c. When will the bank account balance be 0? after 103 mo (8 yr 7 mo)

33. Critical Thinking Complete the following to learn about intercepts and horizontal and vertical lines.

33c. Horiz.: For $y = c$, the y-int. is c, and there is no x-int. Vert.: For $x = k$, the x-int. is k, and there is no y-int.

a. Graph $x = -6$, $x = 1$, and $x = 5$. Find the intercepts.

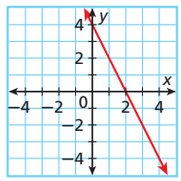
b. Graph $y = -3$, $y = 2$, and $y = 7$. Find the intercepts.

c. Write a rule describing the intercepts of linear equations whose graphs are horizontal and vertical lines.

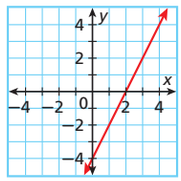
Match each equation with a graph.

34. $-2x - y = 4$ D **35.** $y = 4 - 2x$ A **36.** $2y + 4x = 8$ A **37.** $4x - 2y = 8$ B

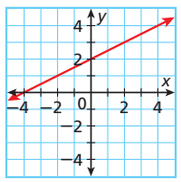
A.



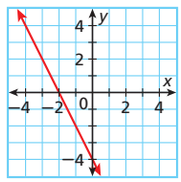
B.



C.



D.



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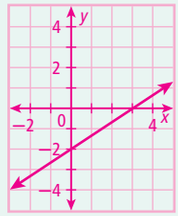
COMMON ERROR ALERT

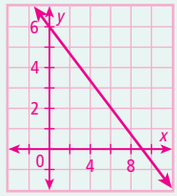
Students may combine intercepts to form a point. In **Exercise 9,** for example, a student might take the x-intercept (5) and the y-intercept (-4) to form the point $(5, -4)$. Remind students that the intercepts give you two separate points: $(5, 0)$ and $(0, -4)$.

Answers

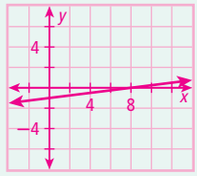
24.



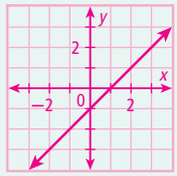
25.



26.



27.

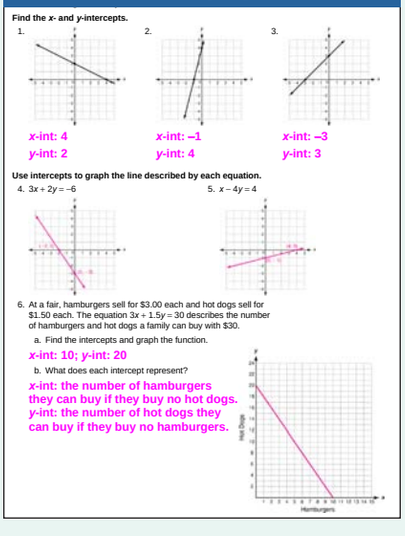


28, 29, 31b, 32a, 33a, 33b. See Additional Answers.

4-2 PRACTICE A

4-2 PRACTICE C

4-2 PRACTICE B



Find the x- and y-intercepts.

1. x-int: 4; y-int: 2
2. x-int: –1; y-int: 4
3. x-int: –3; y-int: 3

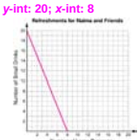
Use intercepts to graph the line described by each equation.

4. $3x + 2y = -6$
5. $x - 4y = 4$
6. At a fair, hamburgers sell for \$3.00 each and hot dogs sell for \$1.50 each. The equation $3x + 1.5y = 30$ describes the number of hamburgers and hot dogs a family can buy with \$30.
   a. Find the intercepts and graph the function.
   x-int: 10; y-int: 20
   b. What does each intercept represent?
   x-int: the number of hamburgers they can buy if they buy no hot dogs. y-int: the number of hot dogs they can buy if they buy no hamburgers.

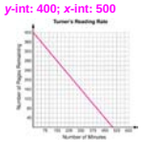
PROBLEM SOLVING

Write the correct answer.

1. Naima has \$40 to spend on refreshments for herself and her friends at the movie theater. The equation $5x + 2y = 40$ describes the number of large popcorns x and small drinks y she can buy. Graph this function and find its intercepts.
   y-int: 20; x-int: 8

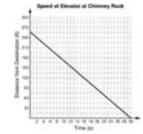


2. Turner is reading a 400-page book. He reads 4 pages every 5 minutes. The number of pages Turner has left to read after x minutes is represented by the function $f(x) = 400 - \frac{4}{5}x$. Graph this function and find its intercepts.
   y-int: 400; x-int: 500



The graph shows the distance of an elevator at Chimney Rock, North Carolina, from its destination as a function of time. Use the graph to answer questions 3–6. Select the best answer.

3. What is the x-intercept of this function?
   A 0 C 258
   B 30 D 300
4. What does the x-intercept represent?
   F the total distance the elevator travels
   G the number of seconds that have passed for any given distance
   H the number of seconds it takes the elevator to reach its destination
   J the distance that the elevator has traveled at any given time
5. What is the y-intercept for this function?
   A 0 C 258
   B 30 D 300



6. What does the y-intercept represent?
   F the total distance the elevator travels
   G the number of seconds that have passed for any given distance
   H the number of seconds it takes the elevator to reach its destination
   J the distance that the elevator has traveled at any given time

CHALLENGE

You can use the **distance formula** to find the distance between two ordered pairs.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

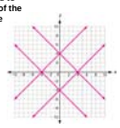
where (x_1, y_1) represents the first ordered pair and (x_2, y_2) represents the second ordered pair.

Find the distance between the ordered pairs. Round your answer to the nearest tenth.

1. (3, 0) and (0, 5) ≈5.8 units
2. (–4, 0) and (0, 2) ≈4.5 units

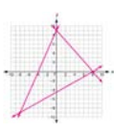
Find the x- and y-intercepts of each equation. Use the intercepts to graph the equations on the same grid. Then find the perimeter of the geometric figure formed by the lines. Round all distances to the nearest tenth.

3. $-x + y = 4$ x-int: –4 y-int: 4
   $-x + y = -4$ x-int: 4 y-int: –4
   $x + y = 4$ x-int: 4 y-int: 4
   $x + y = -4$ x-int: –4 y-int: –4



Perimeter: ≈22.6 units

4. $8y + 9x = 72$ x-int: 8 y-int: 9
   $4y - 9x = 36$ x-int: –4 y-int: 9
   $16y - 9x = -72$ x-int: 8 y-int: –4.5



Perimeter: ≈50.1 units

MULTI-STEP TEST PREP **Exercise 38** involves identifying and interpreting *x*- and *y*-intercepts. This exercise prepares students for the Multi-Step Test Prep.

TEST PREP DOCTOR Students who chose **B** in **Exercise 40** may have chosen the coefficient of *x*. Students who chose **C** found the *y*-intercept instead of the *x*-intercept.

Journal

Have students explain how to find the intercepts of $5x - 8y = 40$.

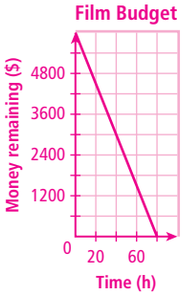
ALTERNATIVE ASSESSMENT

Have students write an equation in standard form, find the intercepts, and then graph the line using the intercepts.

Power Presentations with PowerPoint®

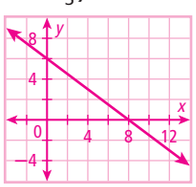
4-2 Lesson Quiz

1. An amateur filmmaker has $6000 to make a film that costs $75/h to produce. The function $f(x) = 6000 - 75x$ gives the amount of money left to make the film after x hours of production. Graph this function and find the intercepts. What does each intercept represent?



x-int.: 80; number of hours when all the money has been spent; *y*-int.: 6000; initial amount of money available

2. Use intercepts to graph the line described by $\frac{1}{4}x = 2 - \frac{1}{3}y$.



Also available online

38. Kristyn rode a stationary bike at the gym. She programmed the timer for 20 minutes. The display counted backward to show how much time remained in her workout. It also showed her mileage.

a. What are the intercepts? ***x*-int.: 20; *y*-int.: 1.75**

b. What do the intercepts represent?
***x*-int.: time remaining when Kristyn started her workout; *y*-int.: total distance Kristyn covered**

Time Remaining (min)	Distance Covered (mi)
20	0
16	0.35
12	0.70
8	1.05
4	1.40
0	1.75

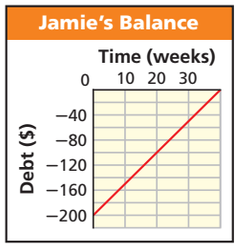
39. Write About It Write a real-world problem that could be modeled by a linear function whose *x*-intercept is 5 and whose *y*-intercept is 60.

TEST PREP

40. Which is the *x*-intercept of $-2x = 9y - 18$?

Ⓐ -9 Ⓑ -2 Ⓒ 2 Ⓓ 9

41. Which of the following situations could be represented by the graph?



Ⓕ Jamie owed her uncle $200. Each week for 40 weeks she paid him $5.

Ⓖ Jamie owed her uncle $200. Each week for 5 weeks she paid him $40.

Ⓗ Jamie owed her uncle $40. Each week for 5 weeks she paid him $200.

Ⓙ Jamie owed her uncle $40. Each week for 200 weeks she paid him $5.

42. Gridded Response What is the *y*-intercept of $60x + 55y = 660$? **12**

CHALLENGE AND EXTEND

Use intercepts to graph the line described by each equation.

43. $\frac{1}{2}x + \frac{1}{5}y = 1$ **44.** $0.5x - 0.2y = 0.75$ **45.** $y = \frac{3}{8}x + 6$

46. For any linear equation $Ax + By = C$, what are the intercepts? ***x*-int.: $\frac{C}{A}$; *y*-int.: $\frac{C}{B}$**

47. Find the intercepts of $22x - 380y = 20{,}900$. Explain how to use the intercepts to determine appropriate scales for the graph.

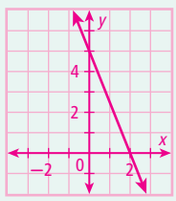
47. *x*-int.: 950; *y*-int.: −55; possible answer: scale on *x*-axis should include numbers from 0 to a number a little greater than 950; scale on *y*-axis should include numbers from a little less than −55 to 0.

Victoria Smith/HMH

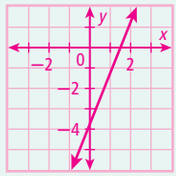
Answers

39. Possible answer: Jen wants to save $60. Each week she will earn $12. The function shows how much money Jen has left to save each week.

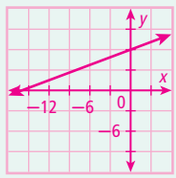
43.



44.



45.



Area in the Coordinate Plane

Lines in the coordinate plane can form the sides of polygons. You can use points on these lines to help you find the areas of these polygons.

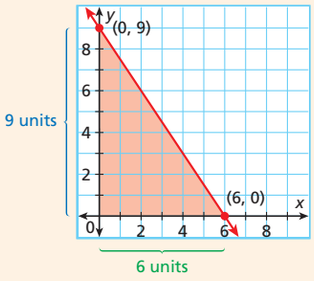
Example

Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 18$.

Step 1 Find the intercepts of $3x + 2y = 18$.

x-intercept:	y-intercept:
$3x + 2y = 18$	$3x + 2y = 18$
$3x + 2(0) = 18$	$3(0) + 2y = 18$
$3x = 18$	$2y = 18$
$x = 6$	$y = 9$

Step 2 Use the intercepts to graph the line. The x-intercept is 6, so plot $(6, 0)$. The y-intercept is 9, so plot $(0, 9)$. Connect with a straight line. Then shade the triangle formed by the line and the axes, as described.



Step 3 Recall that the area of a triangle is given by $A = \frac{1}{2}bh$.

- The length of the base is 6.
- The height is 9.

Step 4 Substitute these values into the formula.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(9)$ *Substitute into the area formula.*

$= \frac{1}{2}(54)$ *Simplify.*

$= 27$

The area of the triangle is 27 square units.

Try This

1. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 12$. **12 sq. units**
2. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $y = 6 - x$. **18 sq. units**
3. Find the area of the polygon formed by the x-axis, the y-axis, the line described by $y = 6$, and the line described by $x = 4$. **24 sq. units**

Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Apply algebra skills to finding areas of geometric figures in the coordinate plane.

Online Edition

Countdown

Teach

Remember

Students review and apply the formulas for the areas of triangles and rectangles.

Discuss with students how identifying the intercepts of the line determines the base and height of the triangle.

Close

Assess

Use the triangle from the example. Have students change the equation of the line so that the area of the triangle increases. Then have them find the area of the new triangle.
Possible answer: $8x + 9y = 72$; 36 square units

Learn It Online
State Resources Online

4-3 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Find rates of change and slopes.

Relate a constant rate of change to the slope of a line.

Technology Lab
In *Technology Lab Activities*

Online Edition
Tutorial Videos, Interactivity

Countdown

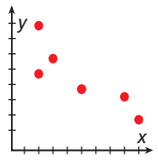
Power Presentations with PowerPoint®

Warm Up

1. Find the *x*- and *y*-intercepts of $2x - 5y = 20$. *x*-int.: 10; *y*-int.: −4

Describe the correlation shown by the scatter plot.

2.



negative

Also available online

Math Humor

Q: Why were the math students getting up and sprinting around the classroom?

A: Their teacher kept saying rise and run.

COMMON CORE

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Learn It Online
State Resources Online

COMMON CORE

4-3 Rate of Change and Slope

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Objectives
Find rates of change and slopes.

Relate a constant rate of change to the slope of a line.

Vocabulary
rate of change
rise
run
slope

Why learn this?
Rates of change can be used to find how quickly costs have increased.

In 1985, the cost of sending a 1-ounce letter was 22 cents. In 1988, the cost was 25 cents. How fast did the cost change from 1985 to 1988? In other words, at what *rate* did the cost change?

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

EXAMPLE 1 ***Consumer Application***

The table shows the cost of mailing a 1-ounce letter in different years. Find the rate of change in cost for each time interval. During which time interval did the cost increase at the greatest rate?

Year	1988	1990	1991	2004	2008
Cost (¢)	25	25	29	37	42

Caution!
A rate of change of 1.25 cents per year for a 4-year period means that the *average* change was 1.25 cents per year. The *actual* change in each year may have been different.

Step 1 Identify the dependent and independent variables.

dependent: cost independent: year

Step 2 Find the rates of change.

1988 to 1990: $\frac{\text{change in cost}}{\text{change in years}} = \frac{25 - 25}{1990 - 1988} = \frac{0}{2} = 0$ *0 cents / year*

1990 to 1991: $\frac{\text{change in cost}}{\text{change in years}} = \frac{29 - 25}{1991 - 1990} = \frac{4}{1} = 4$ *4 cents / year*

1991 to 2004: $\frac{\text{change in cost}}{\text{change in years}} = \frac{37 - 29}{2004 - 1991} = \frac{8}{13} \approx 0.62 \approx$ *0.62 cents / year*

2004 to 2008: $\frac{\text{change in cost}}{\text{change in years}} = \frac{42 - 37}{2008 - 2004} = \frac{5}{4} = 1.25$ *1.25 cents / year*

The cost increased at the greatest rate from 1990 to 1991.

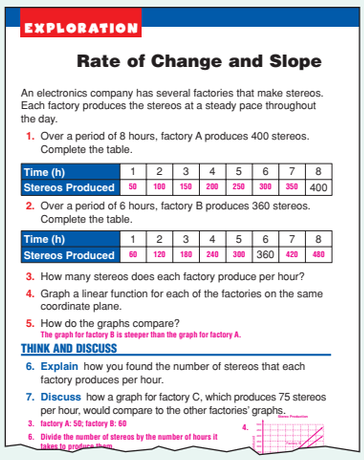
CHECK IT OUT!

1. The table shows the balance of a bank account on different days of the month. Find the rate of change for each time interval. During which time interval did the balance decrease at the greatest rate?

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

1. day 1 to day 6: −53; day 6 to day 16: −7.5; day 16 to day 22: 0; day 22 to day 30: −4.375; from day 1 to day 6

1 Introduce


EXPLORATION

Rate of Change and Slope

An electronics company has several factories that make stereos. Each factory produces the stereos at a steady pace throughout the day.

1. Over a period of 8 hours, factory A produces 400 stereos. Complete the table.

Time (h)	1	2	3	4	5	6	7	8
Stereos Produced	50	100	150	200	250	300	350	400

2. Over a period of 6 hours, factory B produces 360 stereos. Complete the table.

Time (h)	1	2	3	4	5	6	7	8
Stereos Produced	60	120	180	240	300	360	420	480

3. How many stereos does each factory produce per hour?
4. Graph a linear function for each of the factories on the same coordinate plane.
5. How do the graphs compare?
The graph for factory B is steeper than the graph for factory A.

THINK AND DISCUSS

6. **Explain** how you found the number of stereos that each factory produces per hour.
7. **Discuss** how a graph for factory C, which produces 75 stereos per hour, would compare to the other factories' graphs.

3. factory A: 50; factory B: 60
6. Divide the number of stereos by the number of hours it takes to produce them.
4.

Motivate

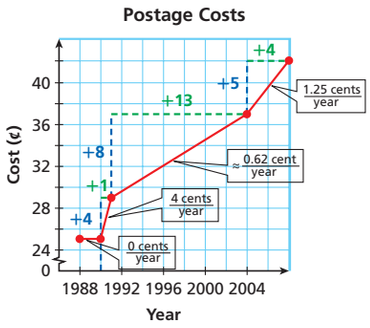
Have students graph a line with *x*-intercept 3 and *y*-intercept 4. On the same grid, have them graph a line with *x*-intercept 5 and *y*-intercept 1. Ask how the lines are the same. Both are slanting in the same direction. Ask how the lines are different. One is steeper than the other.

Tell students that in this section they will learn how to indicate the steepness and direction of a line.

Explorations and answers are provided in your online resources.

EXAMPLE 2 Finding Rates of Change from a Graph

Graph the data from Example 1 and show the rates of change.

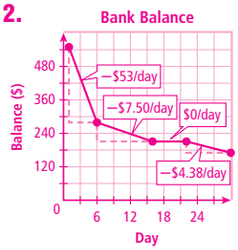


Graph the ordered pairs. The vertical blue segments show the changes in the dependent variable, and the horizontal green segments show the changes in the independent variable.

Notice that the greatest rate of change is represented by the steepest of the red line segments.

Also notice that between 1988 and 1990, when the cost did not change, the red line segment is horizontal.

2.



CHECK IT OUT! **2.** Graph the data from Check It Out Problem 1 and show the rates of change.

If all of the connected segments have the same rate of change, then they all have the same steepness and together form a straight line. The constant rate of change of a nonvertical line is called the *slope* of the line.

Know it! Note

Slope of a Line

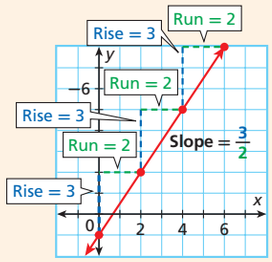
The **rise** is the difference in the ***y*-values** of two points on a line.

The **run** is the difference in the ***x*-values** of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that *y* is the **dependent variable** and *x* is the **independent variable**.)

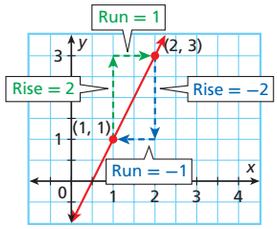


EXAMPLE 3 Finding Slope

Find the slope of the line.

Caution!

Pay attention to the scales on the axes. One square on the grid may not represent 1 unit. In Example 3, each square represents $\frac{1}{2}$ unit.



Begin at one point and count vertically to find the rise.

Then count horizontally to the second point to find the run.

$$\text{slope} = \frac{2}{1} = 2$$

It does not matter which point you start with. The slope is the same.

$$\text{slope} = \frac{-2}{-1} = 2$$

CHECK IT OUT! **3.** Find the slope of the line that contains $(0, -3)$ and $(5, -5)$. $-\frac{2}{5}$

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Additional Examples

Example 1

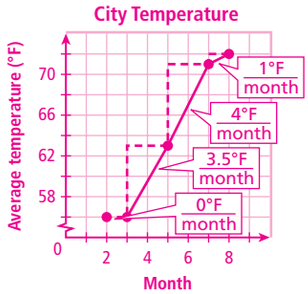
The table shows the average temperature (°F) for five months in a certain city. Find the rate of change in temperature for each time period. During which time period did the temperature increase at the greatest rate?

Month	2	3	5	7	8
Temp. (°F)	56	56	63	71	72

2 to 3: 0; 3 to 5: 3.5; 5 to 7: 4; 7 to 8: 1; from months 5 to 7

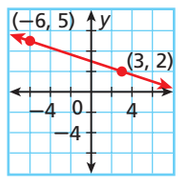
Example 2

Graph the data from Additional Example 1 and show the rates of change.



Example 3

Find the slope of the line.



$-\frac{1}{3}$

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- In a rate of change, the change in which variable goes in the numerator? the denominator?

EXAMPLE 2

- Which variable is dependent? Which is independent?

EXAMPLE 3

- Does it matter which point you start at to find the slope?
- What does a positive slope look like? negative slope? zero slope?

2 Teach

Guided Instruction

Connect the meaning of rate of change to the fact that the independent variable is represented on the horizontal axis, and the dependent variable on the vertical axis. Explain that a positive slope means the rate of change is positive, and a negative slope means the rate of change is negative. Point out that a horizontal line has 0 slope, no change in *y*-values, and 0 rate of change. By contrast, a vertical line has undefined slope and undefined rate of change.

Reaching All Learners

Through Multiple Representations

Show students how to use a "T-Chart" to find the slope.

	x	*y*	
+1	1	1	+3
+1	2	4	+3
	3	7	

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{+3}{+1} = 3$$

This method works whether the ordered pairs are given in a table or a list, or selected from a graph.

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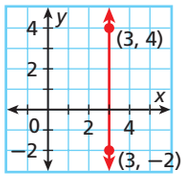
Additional Examples

Example 4

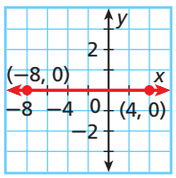
Find the slope of each line.

A.



undefined

B.

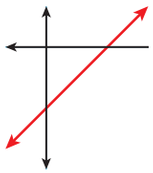


0

Example 5

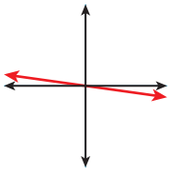
Tell whether the slope of each line is positive, negative, zero, or undefined.

A.



positive

B.



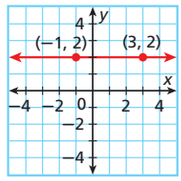
negative

Also available online

EXAMPLE 4 Finding Slopes of Horizontal and Vertical Lines

Find the slope of each line.

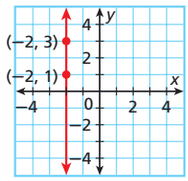
A



$\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0$

The slope is 0.

B

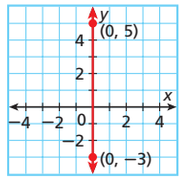


$\frac{\text{rise}}{\text{run}} = \frac{2}{0}$ *You cannot divide by 0.*

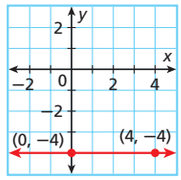
The slope is undefined.

CHECK IT OUT! **Find the slope of each line.**

4a. undef.

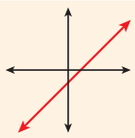
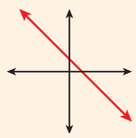
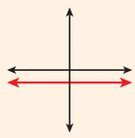
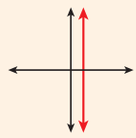


4b. 0



As shown in the previous examples, slope can be positive, negative, zero, or undefined. You can tell which of these is the case by looking at the graph of a line—you do not need to calculate the slope.

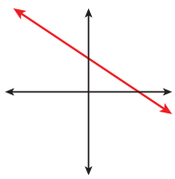
Know it! Note

Positive Slope	Negative Slope	Zero Slope	Undefined Slope
			
Line rises from left to right.	Line falls from left to right.	Horizontal line	Vertical line

EXAMPLE 5 Describing Slope

Tell whether the slope of each line is positive, negative, zero, or undefined.

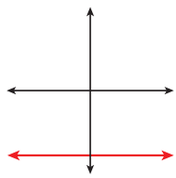
A



The line falls from left to right.

The slope is negative.

B



The line is horizontal.

The slope is 0.

Questioning Strategies

EXAMPLE 4

- Why is the slope of a vertical line undefined?

EXAMPLE 5

- Why don't you need points to determine whether the slope is positive, negative, zero, or undefined?

Reaching All Learners

Through Cognitive Strategies

To help students remember the slopes of horizontal and vertical lines, remind them to think about which line has a rise of 0. Since a horizontal line does not rise at all, the rise is 0, and the slope is 0.

Through Auditory Cues

To help students interpret the meanings of different slopes, emphasize reading the graph *from left to right.*

Positive slope → *y*-values increase ***from left to right.***

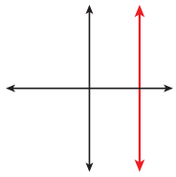
Negative slope → *y*-values decrease ***from left to right.***

Zero slope → *y*-values have zero change ***from left to right.***

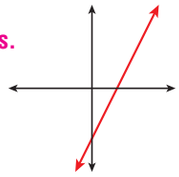
Undefined slope → There is ***NO left to right;*** the line is vertical.

CHECK IT OUT! Tell whether the slope of each line is positive, negative, zero, or undefined.

5a. undef.

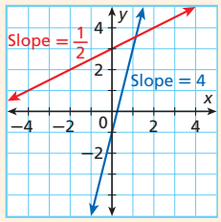


5b. pos.



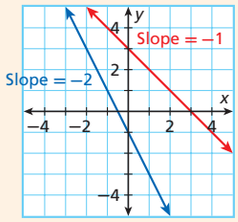
A line's slope is a measure of its steepness. Some lines are steeper than others. As the absolute value of the slope increases, the line becomes steeper. As the absolute value of the slope decreases, the line becomes less steep.

Comparing Slopes



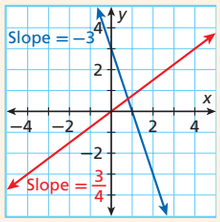
The line with slope 4 is steeper than the line with slope $\frac{1}{2}$.

$|4| > \left|\frac{1}{2}\right|$



The line with slope −2 is steeper than the line with slope −1.

$|-2| > |-1|$



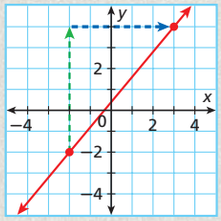
The line with slope −3 is steeper than the line with slope $\frac{3}{4}$.

$|-3| > \left|\frac{3}{4}\right|$

MATHEMATICAL PRACTICES

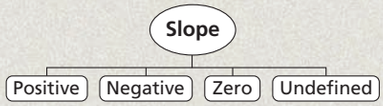
THINK AND DISCUSS

1. What is the rise shown in the graph? What is the run? What is the slope?



2. The rate of change of the profits of a company over one year is negative. How have the profits of the company changed over that year?
3. Would you rather climb a hill with a slope of 4 or a hill with a slope of $\frac{5}{2}$? Explain your answer.
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a line whose slope matches the given description.

Know it! Note



COMMON ERROR ALERT

Students may write 0 for the slope of a vertical line, thinking that "0" and "undefined" are the same. Remind students that 0 is a real number; it is not undefined.

Teaching Tip **Reading Math** The words *gradient, slant,* and *incline* have meanings similar to *slope.*

ENGLISH LANGUAGE LEARNERS

Teaching Tip **Inclusion** Point out that in the slope ratio, the rise (change in *y*-values) is recorded before the run (change in *x*-values). This is the opposite of students' understanding from working in the coordinate plane. Everything up until now has been *x*-values (horizontal) first and then *y*-values (vertical) second. Direct students' attention to this difference.

3 Close

Summarize

Ask students to give descriptions of lines with the following slopes:

1. $m = 0$ horizontal line
2. $m = \frac{1}{50}$ line rising from left to right, not steep
3. $m = -10$ line falling from left to right, steep
4. $m = \frac{2}{0}$ vertical line

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. 6 units; 5 units; $\frac{6}{5}$
2. decreased
3. Possible answer: $\frac{5}{2}$, because it is less steep
4. See Additional Answers.

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
- **Basic** 12–15, 24, 26
- **Average** 12–15, 23, 24, 26, 27, 32
- **Advanced** 12–15, 23, 24, 27, 32, 33

If you finished Examples **1–5**
- **Basic** 10–20, 22, 27, 29–31
- **Average** 12–32
- **Advanced** 12–25, 27–33

Homework Quick Check
Quickly check key concepts.
Exercises: 12, 13, 14, 16, 18, 24

Teaching Tip **Inclusion** Emphasize the difference between non-constant rates of change and constant rates of change (slope). The graph of **Exercise 3** will show different slopes, indicating different rates of change. The graph of **Exercise 4** shows the same slope, so the rate of change is constant.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 2–3, 12–13, 20, 22–24, 26–27, 32–33

Construct viable arguments and critique the reasoning of others. Exercises 21, 25

Use appropriate tools strategically. Exercises 4–7, 14–17, 28

Attend to precision. Exercise 27

Look for and make use of structure. Exercises 8–11, 18–19

Learn It Online
State Resources Online

4-3 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** The *slope* of any nonvertical line is ___?___. (*positive* or *constant*) constant

SEE EXAMPLE 1

2. The table shows the volume of gasoline in a gas tank at different times. Find the rate of change for each time interval. During which time interval did the volume decrease at the greatest rate?

Time (h)	0	1	3	6	7
Volume (gal)	12	9	5	1	1

hour 0 to hour 1: −3; hour 1 to hour 3: −2; hour 3 to hour 6: $-\frac{4}{3}$; hour 6 to hour 7: 0; from hour 0 to hour 1

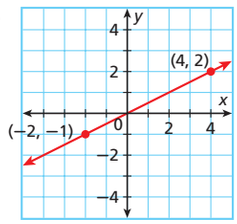
SEE EXAMPLE 2

3. The table shows a person's heart rate over time. Graph the data and show the rates of change.

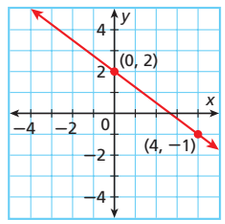
Time (min)	0	2	5	7	10
Heart Rate (beats/min)	64	92	146	84	64

Find the slope of each line.

SEE EXAMPLE 3

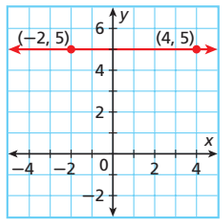
4. 

$\frac{1}{2}$

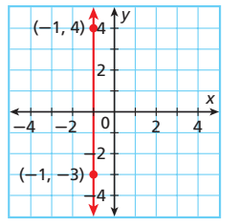
5. 

$-\frac{3}{4}$

SEE EXAMPLE 4

6. 

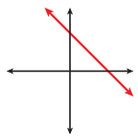
0

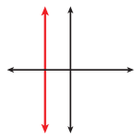
7. 

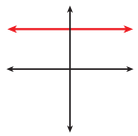
undef.

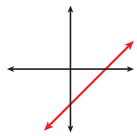
SEE EXAMPLE 5

Tell whether the slope of each line is positive, negative, zero, or undefined.

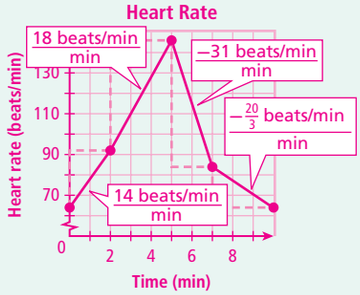
8. 
neg.

9. 
undef.

10. 
zero

11. 
pos.

Answers

3.


PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12	1
13	2
14–15	3
16–17	4
18–19	5

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

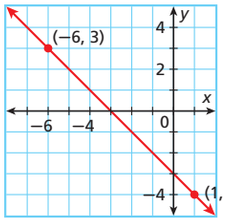
12. The table shows the length of a baby at different ages. Find the rate of change for each time interval. Round your answers to the nearest tenth. During which time interval did the baby have the greatest growth rate? **month 3 to month 9: 0.7; month 9 to month 18: 0.5; month 18 to month 26: 0.4; month 26 to month 33: 0.3; from 3 months to 9 months**

Age (mo)	3	9	18	26	33
Length (in.)	23.5	27.5	31.6	34.5	36.7

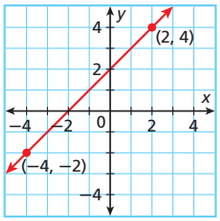
13. The table shows the distance of an elevator from the ground floor at different times. Graph the data and show the rates of change.

Time (s)	0	15	23	30	35
Distance (m)	30	70	0	45	60

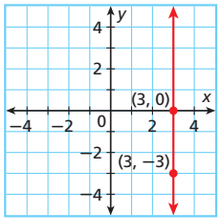
Find the slope of each line.

14. 

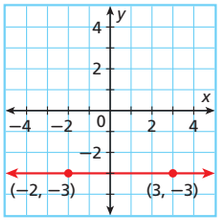
−1

15. 

1

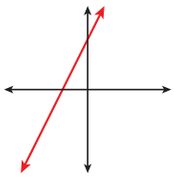
16. 

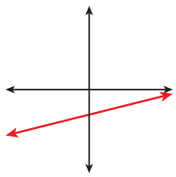
undef.

17. 

0

Tell whether the slope of each line is positive, negative, zero, or undefined.

18. 
pos.

19. 
pos.

Travel

The Incline Railway's climb up Lookout Mountain has been called "America's Most Amazing Mile." A round-trip on the railway lasts about 1.5 hours.

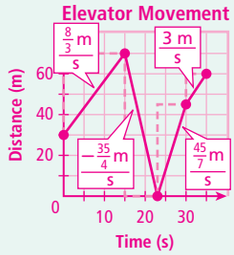
© Dave G. Houser/CORBIS

20. Travel The Lookout Mountain Incline Railway in Chattanooga, Tennessee, is the steepest passenger railway in the world. A section of the railway has a slope of about 0.73. In this section, a vertical change of 1 unit corresponds to a horizontal change of what length? Round your answer to the nearest hundredth. **1.37**

21. Critical Thinking In Lesson 5-1, you learned that in a linear function, a constant change in x corresponds to a constant change in y. How is this related to slope?

Answers

13.

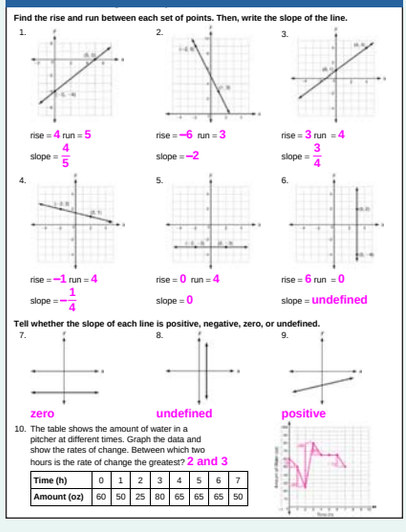


21. Possible answer: Slope is the ratio of change in y to change in x, and, for a line, it is always constant.

PRACTICE A

PRACTICE C

PRACTICE B


Find the rise and run between each set of points. Then, write the slope of the line.

1. rise = 4 run = 5 slope = 4/5
2. rise = −6 run = 3 slope = −2
3. rise = 3 run = 4 slope = 3/4
4. rise = −1 run = 4 slope = −1/4
5. rise = 0 run = 4 slope = 0
6. rise = 6 run = 0 slope = undefined

Tell whether the slope of each line is positive, negative, zero, or undefined.

7. zero
8. undefined
9. positive

10. The table shows the amount of water in a pitcher at different times. Graph the data and show the rates of change. Between which two hours is the rate of change the greatest? 2 and 3

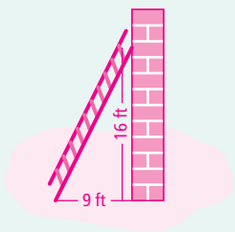
Time (h)	0	1	2	3	4	5	6	7
Amount (oz)	60	50	25	80	65	65	65	50

MULTI-STEP TEST PREP **Exercise 22** involves finding the slope from a graph and describing the rate of change. This exercise prepares students for the Multi-Step Test Prep.

Data Collection To help students complete **Exercise 28,** see *Technology Lab Activities*.

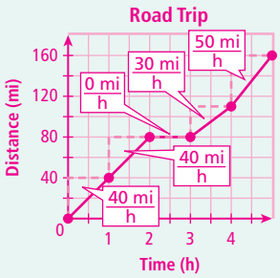
Answers

24a.



25. Possible answer: The slope of a horizontal line will always be 0 because the *y*-coordinates of any 2 points will be the same. Therefore, the numerator in the slope formula will always be 0. The slope of a vertical line will always be undefined because the *x*-coordinates of any 2 points will be the same. Therefore, the denominator in the slope formula will always be 0. Since you can't divide by 0, the slope will always be undefined.

26a.



28a. Walk toward or away from the motion detector at a constant rate. A line has constant slope, and in this case slope represents $\frac{\text{distance}}{\text{time}}$, or rate. So keeping the rate constant will result in a line.

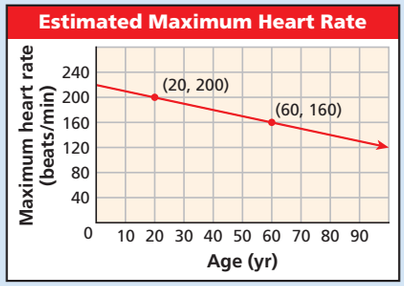
b. Positive slope: walk away from the detector; negative slope: walk toward the detector.

c. Stand still—as time passes, your distance from the detector does not change. This graph is a horizontal line.

MULTI-STEP TEST PREP

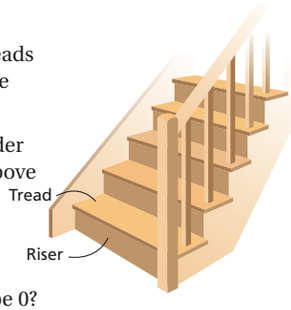
22. a. The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the slope. **−1**

b. Describe the rate of change in this situation. **For each yr that a person ages, their max. heart rate decreases by 1 beat/min.**



23. $\frac{8.5}{9} \approx 0.9444$

23. Construction Most staircases in use today have 9-inch treads and $8\frac{1}{2}$-inch risers. What is the slope of a staircase with these measurements?



24. A ladder is leaned against a building. The bottom of the ladder is 9 feet from the building. The top of the ladder is 16 feet above the ground.

a. Draw a diagram to represent this situation.

b. What is the slope of the ladder? $\frac{16}{9}$

25. Write About It Why will the slope of any horizontal line be 0? Why will the slope of any vertical line be undefined?

26. The table shows the distance traveled by a car during a five-hour road trip.

Time (h)	0	1	2	3	4	5
Distance (mi)	0	40	80	80	110	160

a. Graph the data and show the rates of change.

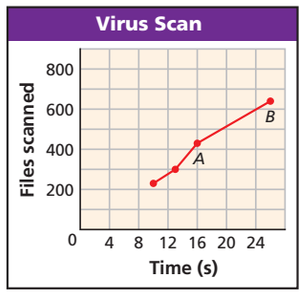
b. The rate of change represents the average speed. During which hour was the car's average speed the greatest? **the fifth**

27a. Possible answer: (16, 420)

b. Possible answer: (26, 650)

c. Possible answer: 23

27. Estimation The graph shows the number of files scanned by a computer virus detection program over time.



a. Estimate the coordinates of point *A*.

b. Estimate the coordinates of point *B*.

c. Use your answers from parts **a** and **b** to estimate the rate of change (in files per second) between points *A* and *B*.

28. Data Collection Use a graphing calculator and a motion detector for the following. Set the equipment so that the graph shows distance on the *y*-axis and time on the *x*-axis.

a. Experiment with walking in front of the motion detector. How must you walk to graph a straight line? Explain.

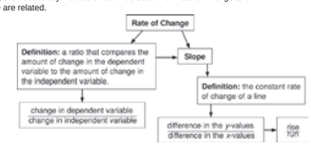
b. Describe what you must do differently to graph a line with a positive slope vs. a line with a negative slope.

c. How can you graph a line with slope 0? Explain.

Victoria Smith/HMH

READING STRATEGIES

Study the vocabulary web below to understand how rate of change and slope are related.



The table shows the temperatures (in °F) at one location at different times.

Time	8 am	11 am	1 pm	5 pm
Temperature	68	80	84	88

1. Which is the dependent variable? temperature
2. Which is the independent variable? time
3. Calculate the rate of change for each time interval.
   8 am to 11 am: $\frac{12}{3} = 4$ 11 am to 1 pm: $\frac{4}{2} = 2$ 1 pm to 5 pm: $\frac{4}{4} = 1$
4. In everyday life, *rise* means to move in an upward direction. How can this help you remember the math definition?
   upward is vertical and the *y*-axis is the vertical axis.
5. How can you use the everyday meaning of *run* to help you remember the math definition?
   people run horizontally and the *x*-axis is the horizontal axis.
6. Find the slope of the line.

$-\frac{2}{3}$

RETEACH

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

The table shows the average retail price of peanut butter from 1986 to 1997. Find the rate of change in cost for each time interval. During which time interval did the cost increase at the greatest rate?

Year	1986	1987	1989	1992	1997
Cost per lb ($)	1.60	1.80	1.81	1.94	1.78

Step 1: Identify independent and dependent variables.
Year is independent. *Cost* is dependent.

Step 2: Find the rates of change.

1986 to 1987: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.80-1.60}{1987-1986} = \frac{0.20}{1} = 0.2$ greatest rate of change

1987 to 1989: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.81-1.80}{1989-1987} = \frac{0.01}{2} = 0.005$

1989 to 1992: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.94-1.81}{1992-1989} = \frac{0.13}{3} = 0.043$

1992 to 1997: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.78-1.94}{1997-1992} = \frac{-0.16}{5} = -0.032$ This rate of change is negative. The price went down during this time period.

The cost increased at the greatest rate from 1986 to 1987.

The table shows the average retail price of cherries from 1986 to 1991. Find the rate of change in cost for each time interval.

Year	1986	1988	1989	1991
Cost per lb ($)	1.27	1.63	1.15	2.26

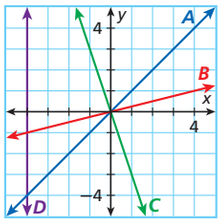
1. 1986 to 1988: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.63-1.27}{1988-1986} = \frac{0.36}{2} = 0.18$
2. 1988 to 1989: $\frac{\text{change in cost}}{\text{change in years}} = \frac{1.15-1.63}{1989-1988} = \frac{-0.48}{1} = -0.48$
3. 1989 to 1991: $\frac{\text{change in cost}}{\text{change in years}} = \frac{2.26-1.5}{1991-1989} = \frac{1.11}{2} = 0.555$
4. Which time interval showed the greatest rate of change? 1989 to 1991
5. Was the rate of change ever negative? If so, when? yes; 1988 to 1989

TEST PREP

29. The slope of which line has the greatest absolute value?

Ⓐ line *A* Ⓒ line *C*

Ⓑ line *B* Ⓓ line *D*

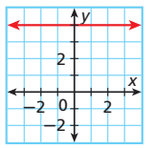


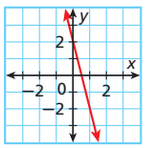
30. For which line is the run equal to 0?

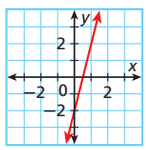
Ⓐ line *A* Ⓒ line *C*

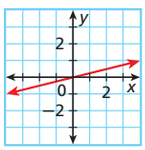
Ⓑ line *B* Ⓓ line *D*

31. Which line has a slope of 4?

Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

CHALLENGE AND EXTEND

32. Recreation Tara and Jade are hiking up a hill. Each has a different stride. The run for Tara's stride is 32 inches, and the rise is 8 inches. The run for Jade's stride is 36 inches. What is the rise of Jade's stride? **9 in.**

33. Economics The table shows cost in dollars charged by an electric company for various amounts of energy in kilowatt-hours.

Energy (kWh)	0	200	400	600	1000	2000
Cost ($)	3	3	31	59	115	150

a. Graph the data and show the rates of change.

b. Compare the rates of change for each interval. Are they all the same? Explain.

c. What do the rates of change represent?

d. Describe in words the electric company's billing plan.

TEST PREP DOCTOR In **Exercise 31,** choices **F** and **H** can be eliminated immediately—the line in choice **F** has zero slope, and the line in choice **H** has a negative slope.

Journal

Have students compare a line with a slope of 3 and a line with a slope of $\frac{1}{3}$. Ask them to explain how they are alike, how they are different, and which line is steeper.

ALTERNATIVE ASSESSMENT

Have students explain how to tell whether the slope of a line is positive, negative, zero, or undefined simply by looking at the graph. They should include sketches in their answer.

Power Presentations with PowerPoint®

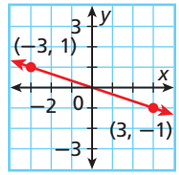
4-3 Lesson Quiz

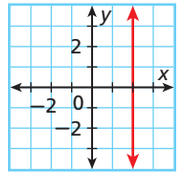
1. The table shows the number of bikes made by a company for certain days. Find the rate of change for each time period. During which time period did the number of bikes increase at the fastest rate?

Day	1	2	5	7	11
Bikes	32	35	47	47	61

1 to 2: 3; 2 to 5: 4; 5 to 7: 0; 7 to 11: 3.5; from days 2 to 5

Find the slope of each line.

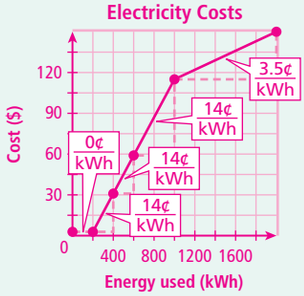
2.  $-\frac{1}{3}$

3.  undefined

Also available online

Answers

33a.



b. 0 kWh to 200 kWh: 0; 200 kWh to 400 kWh: 0.14; 400 kWh to 600 kWh: 0.14; 600 kWh to 1000 kWh: 0.14; 1000 kWh to 2000 kWh: 0.035; the rates of change for 200 kWh to 400 kWh, 400 kWh to 600 kWh, and 600 kWh to 1000 kWh are equivalent.

c. the cost per kilowatt-hour

d. Up to 200 kWh costs $3.00. Between 200 and 1000 kWh costs $0.14 per kWh. Between 1000 and 2000 kWh costs $0.035 per kWh.

Organizer

Use with the lesson Rate of Change and Slope

Pacing:
Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Explore the relationship between constant change and the slope of a line.

Online Edition

Countdown

Lab Resources

Algebra Lab Activities
Lab Recording Sheet

Teach

Discuss

Encourage students to identify the independent and dependent variables for each situation.

Alternative Approach

Have students write the function rule for each scenario and compare the number multiplied by the independent variable with the amount of constant change.

Answers to *Activity 1*

2. The number of books Janice reads per week; the number of books read increases by 2 each week.

MATHEMATICAL PRACTICES

Look for and express regularity in repeated reasoning.

COMMON CORE

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Learn It Online
State Resources Online

4-3

Explore Constant Changes

Use with Rate of Change and Slope

There are many real-life situations in which the amount of change is constant. In these activities, you will explore what happens when

- a quantity increases by a constant amount.
- a quantity decreases by a constant amount.

MATHEMATICAL PRACTICES **Look for and express regularity in repeated reasoning.**

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Activity 1

Janice has read 7 books for her summer reading club. She plans to read 2 books each week for the rest of the summer. The table shows the total number of books that Janice will have read after different numbers of weeks have passed.

Janice's Summer Reading	
Week	Total Books Read
0	7
1	9
2	11
3	13
4	15
5	17

1. What number is added to the number of books in each row to get the number of books in the next row? 2
2. What does your answer to Problem 1 represent in Janice's situation? Describe the meaning of the constant change.
3. Graph the ordered pairs from the table. Describe how the points are related.
4. Look again at your answer to Problem 1. Explain how this number affects your graph.

Try This

At a particular college, a full-time student must take at least 12 credit hours per semester and may take up to 18 credit hours per semester. Tuition costs $200 per credit hour.

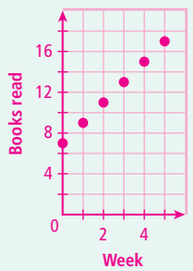
Tuition Costs	
Credit Hours	Cost ($)
12	2400
13	2600
14	2800
15	3000
16	3200
17	3400
18	3600

1. Copy and complete the table by using the information above.
2. What number is added to the cost in each row to get the cost in the next row? 200
3. What does your answer to Problem 2 above represent in the situation? Describe the meaning of the constant change.
4. Graph the ordered pairs from the table. Describe how the points are related.
5. Look again at your answer to Problem 2. Explain how this number affects your graph.
6. Compare your graphs from Activity 1 and Problem 4. How are they alike? How are they different?
7. **Make a Conjecture** Describe the graph of any situation that involves repeated addition of a positive number. Why do you think your description is correct?

Possible answer: It will be a line that ascends from left to right because the *y*-values will keep increasing by the same amount.

Answers to *Activity 1*

3. Janice's Summer Reading

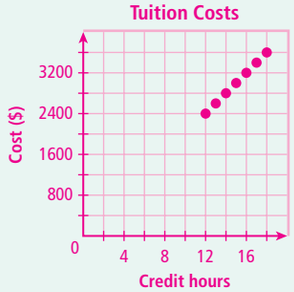


The points lie on a line that ascends from left to right.

4. Possible answer: The *y*-coordinate of each point is 2 greater than that of the previous point.

Answers to *Try This*

3. The cost of one credit hour; each additional credit hour increases the cost of tuition by $200.
4. Tuition Costs



The points lie on a line that ascends from left to right.

5. Possible answer: The *y*-coordinate of each point is 200 greater than that of the previous point.
6. Possible answer: They are both points on lines that ascend from left to right. The tuition graph forms a steeper line.

Activity 2

An airplane is 3000 miles from its destination. The plane is traveling at a rate of 540 miles per hour. The table shows how far the plane is from its destination after various amounts of time have passed.

Airplane's Distance	
Time (h)	Distance to Destination (mi)
0	3000
1	2460
2	1920
3	1380
4	840

1. What number is subtracted from the distance in each row to get the distance in the next row? **540**
2. What does your answer to Problem 1 represent in the situation? Describe the meaning of the constant change.
3. Graph the ordered pairs from the table. Describe how the points are related.
4. Look again at your answer to Problem 1. Explain how this number affects your graph.

Try This

A television game show begins with 20 contestants. Each week, the players vote 2 contestants off the show.

Game Show	
Week	Contestants Remaining
0	20
1	18
2	16
3	14
4	12
5	10
6	8

8. Copy and complete the table by using the information above.
9. What number is subtracted from the number of contestants in each row to get the number of contestants in the next row? **2**
10. What does your answer to Problem 9 represent in the situation? Describe the meaning of the constant change.
11. Graph the ordered pairs from the table. Describe how the points are related.
12. Look again at your answer to Problem 9. Explain how this number affects your graph.
13. Compare your graphs from Activity 2 and Problem 11. How are they alike? How are they different?
14. **Make a Conjecture** Describe the graph of any situation that involves repeated subtraction of a positive number. Why do you think your description is correct?
15. Compare your two graphs from Activity 1 with your two graphs from Activity 2. How are they alike? How are they different?
16. **Make a Conjecture** How are graphs of situations involving repeated subtraction different from graphs of situations involving repeated addition? Explain your answer.

14. Possible answer: It will be a line that descends from left to right because you will keep decreasing the *y*-values by the same amount.

15. Possible answer: They both form lines. In Act. 1, the lines were ascending from left to right. In Act. 2, the lines were descending from left to right.

16. Possible answer: When situations involve repeated subt., the line is descending. When situations involve repeated add., the line is ascending.

Teacher to Teacher

Jennifer J. Southers
Piedmont, SC

When I introduce slope, I use a motion detector to motivate the lesson. I have a student walk at a normal pace toward the detector while the class watches the graph appear on the overhead panel. We discuss how the axes are labeled (distance-time). Students make and test conjectures about how to make the line more or less steep (walk faster or slower) and about how to change the direction of the graph (walk toward or away from the detector).

We discuss the units of the slope (m/s) and determine that this is speed, a rate of change. This also helps students remember that the slope of a horizontal line is zero (standing still, having zero speed), and that a vertical line has undefined slope, since it cannot be created by the detector. By starting in this fashion, I can always refer back to the motion detector activity as students continue to learn about slope.

Close

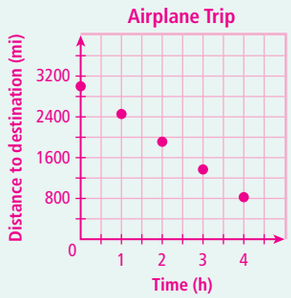
Key Concept

If a situation involves a quantity that increases or decreases by a constant amount, the data points will form a straight line. The steepness of the line is determined by the amount of increase or decrease.

Assessment

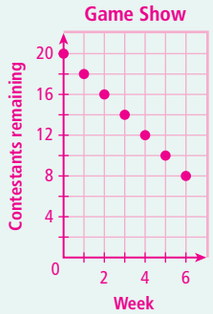
Journal Create a word problem involving a situation that uses repeated addition or subtraction. Write at least five ordered pairs and graph the line. Explain how the graph is related to the situation.

Answers to *Activity 2*

2. The plane's speed; the distance to the destination is decreasing by 540 mi each hour.
3. 

The points lie on a line that descends from left to right.
4. Possible answer: The *y*-coordinate of each point is 540 less than that of the previous point.

Answers to *Try This*

10. The number of contestants voted off each week; the total number of contestants is decreasing by 2 each week.
11. 

The points lie on a line that descends from left to right.
12. Possible answer: The *y*-coordinate of each point is 2 less than that of the previous point.
13. Possible answer: Both are points on lines that descend from left to right. The airplane graph is steeper.

4-4 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Find slope by using the slope formula.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Add or subtract.

1. $4 + (-6)$ -2 **2.** $-3 + 5$ 2

3. $-7 - 7$ -14 **4.** $2 - (-1)$ 3

Find the *x*- and *y*-intercepts.

5. $x + 2y = 8$ *x*-int.: 8; *y*-int.: 4

6. $3x + 5y = -15$ *x*-int.: -5; *y*-int.: -3

Also available online

Math Humor

A baker told his assistant to halve the filling recipe for a cake. When the baker asked for the filling, the assistant replied, "But I ate it!"

"Why did you do that?" demanded the baker.

"You told me to have it!"

COMMON CORE

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Learn It Online
State Resources Online

COMMON CORE

4-4 The Slope Formula

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Objective
Find slope by using the slope formula.

Why learn this?
You can use the slope formula to find how quickly a quantity, such as the amount of water in a reservoir, is changing. (See Example 3.)

In a previous lesson, slope was described as the constant rate of change of a line. You saw how to find the slope of a line by using its graph.

There is also a formula you can use to find the slope of a line, which is usually represented by the letter m. To use this formula, you need the coordinates of two different points on the line.

Know it! Note

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in *y*-values to the difference in *x*-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7$.

EXAMPLE 1 **Finding Slope by Using the Slope Formula**

Find the slope of the line that contains $(4, -2)$ and $(-1, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

$$= \frac{2 - (-2)}{-1 - 4}$$ *Substitute $(4, -2)$ for (x_1, y_1) and $(-1, 2)$ for (x_2, y_2).*

$$= \frac{4}{-5}$$ *Simplify.*

$$= -\frac{4}{5}$$

The slope of the line that contains $(4, -2)$ and $(-1, 2)$ is $-\frac{4}{5}$.

Reading Math

The small numbers to the bottom right of the variables are called subscripts. Read x_1 as "*x* sub one" and y_2 as "*y* sub two."

CHECK IT OUT!

1a. Find the slope of the line that contains $(-2, -2)$ and $(7, -2)$. 0

1b. Find the slope of the line that contains $(5, -7)$ and $(6, -4)$. 3

1c. Find the slope of the line that contains $\left(\frac{3}{4}, \frac{7}{5}\right)$ and $\left(\frac{1}{4}, \frac{2}{5}\right)$. 2

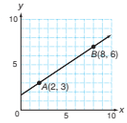
© Patrick Eden/Alamy Photos

1 Introduce

EXPLORATION

The Slope Formula

In this Exploration, you will investigate a shortcut for finding the slope of a line.



1. What is the rise from point *A* to point *B*? 3
2. What is the run from point *A* to point *B*? 6
3. What is the slope of the line? $\frac{1}{2}$
4. Explain how you could have found the rise by using only the *y*-coordinates of the two points, without seeing the graph. Subtract the *y*-coordinates (6 − 3 = 3).
5. Explain how you could have found the run by using only the *x*-coordinates of the two points, without seeing the graph. Subtract the *x*-coordinates (8 − 2 = 6).

THINK AND DISCUSS

6. **Describe** a general method you can use to find the slope of a line if you are given the coordinates of two points on the line. Subtract the *y*-coordinates to find the rise; subtract the *x*-coordinates to find the run; then divide the rise by the run.
7. **Explain** how you could use your method to find the slope of the line that contains the points (5, 8) and (10, 18). rise = 18 − 8 = 10; run = 10 − 5 = 5; slope = $\frac{10}{5} = 2$

Motivate

Display the graph of a line (not horizontal or vertical) and plot and label two points whose coordinates are fractions or decimals. Ask students how they would find the slope. Possible answer: Count the rise and run and then find $\frac{\text{rise}}{\text{run}}$. Because the coordinates are not integers, counting rise and run is more difficult, and an error is likely. Tell students that in this lesson, they will learn a formula to find the slope without counting.

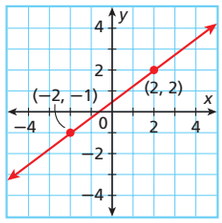
Explorations and answers are provided in your online resources.

Sometimes you are not given two points to use in the formula. You might have to choose two points from a graph or a table.

EXAMPLE 2 Finding Slope from Graphs and Tables

Each graph or table shows a linear relationship. Find the slope.

A



Let $(2, 2)$ be (x_1, y_1) and $(-2, -1)$ be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

$$= \frac{-1 - 2}{-2 - 2}$$ *Substitute $(2, 2)$ for (x_1, y_1) and $(-2, -1)$ for (x_2, y_2).*

$$= \frac{-3}{-4}$$ *Simplify.*

$$= \frac{3}{4}$$

B

x	2	2	2	2
y	0	1	3	5

Step 1 Choose any two points from the table. Let $(2, 0)$ be (x_1, y_1) and $(2, 3)$ be (x_2, y_2).

Step 2 Use the slope formula.

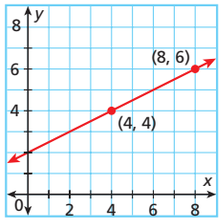
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Use the slope formula.*

$$= \frac{3 - 0}{2 - 2}$$ *Substitute $(2, 0)$ for (x_1, y_1) and $(2, 3)$ for (x_2, y_2).*

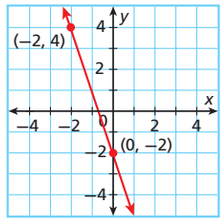
$$= \frac{3}{0}$$ *Simplify.*

The slope is undefined.

CHECK IT OUT! **Each graph or table shows a linear relationship. Find the slope.**

2a. 

$\frac{1}{2}$

2b. 

-3

2c.

x	0	2	5	6
y	1	5	11	13

2

2d.

x	−2	0	2	4
y	3	0	−3	−6

$-\frac{3}{2}$

Remember that slope is a rate of change. In real-world problems, finding the slope can give you information about how a quantity is changing.

COMMON ERROR ALERT

When using the slope formula, students may subtract the *x*- and *y*-values in different orders. Encourage students to label their ordered pairs *first* and *second* so that the *x*- and *y*-values are always subtracted in the same order.

Power Presentations with PowerPoint®

Additional Examples

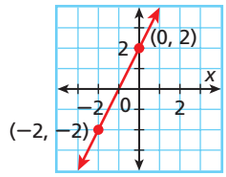
Example 1

Find the slope of the line that contains (2, 5) and (8, 1). $-\frac{2}{3}$

Example 2

Each graph or table shows a linear relationship. Find the slope.

A. 2



B.

x	−2	−1	0	1
y	5	3	1	−1

−2

Also available online

2 Teach

Guided Instruction

Be sure students understand slope as the ratio of rise to run. Be sure that students can see that both methods—using the slope formula and using a graph to count rise and run—yield the same result.

Teaching Tip **Multiple Representations** It is important for students to realize that the slope of a line is always the same, no matter which two points on the line are used to determine the slope. Use examples to demonstrate this.

Reaching All Learners *Through Cooperative Learning*

Divide the class into groups of 3–5 students. Each group will need four number cubes. These can be found in the Manipulatives Kit (MK). One member of the group rolls the number cubes. Each member then uses the rolled numbers to create two ordered pairs and calculates the slope of the line between these two points. The student who finds the steepest slope wins the round and receives one point. (Note that a negative slope may be steeper than a positive one.) Play until one student reaches five points.

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- If a line has slope 0, what will be true about the *y*-coordinates of any two points on that line?
- If a line has undefined slope, what will be true about the *x*-coordinates of any two points on that line?

EXAMPLE 2

- When you are given more than one point, how do you know which points to use in the slope formula?
- How could you check your answers?

Teaching Tip **Visual** For problems like those in **Example 1,** students can check that their answer is reasonable by plotting the two points and connecting them with a line. They can then check that the direction of the line matches the sign of the slope.

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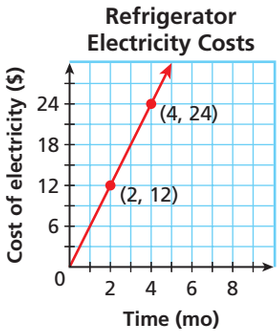
Additional Examples

Example 3

The graph shows the average electricity costs (in dollars) for operating a refrigerator for several months. Find the slope of the line. Then tell what the slope represents.



slope = 6; cost to run refrigerator is $6/mo

Example 4

Find the slope of the line described by $4x - 2y = 16$. 2

Also available online

Questioning Strategies

EXAMPLE 3

- What kind of slope represents a negative rate of change? a positive rate of change?
- What does time 0 represent?

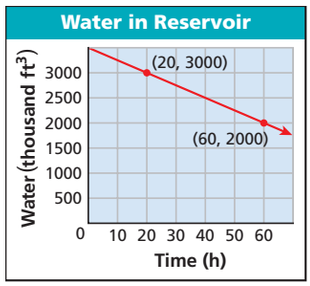
EXAMPLE 4

- Do you have to find the *x*- and *y*-intercepts to find the slope? Explain why or why not.
- How could you solve this problem by graphing?

Teaching Tip **Math Background** Point out to students that the slope formula is the change in *y* divided by the corresponding change in *x*, which can also be represented with the Greek letter Δ (delta): $\frac{\Delta y}{\Delta x}$, which is read "delta *y* over delta *x*."

EXAMPLE 3 Environmental Science Application

The graph shows how much water is in a reservoir at different times. Find the slope of the line. Then tell what the slope represents.



Step 1 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2000 - 3000}{60 - 20}$$
$$= \frac{-1000}{40} = -25$$

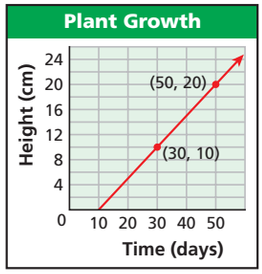
Step 2 Tell what the slope represents.

In this situation, *y* represents **volume of water** and *x* represents **time**.

So slope represents $\frac{\text{change in volume}}{\text{change in time}}$ in units of $\frac{\text{thousands of cubic feet}}{\text{hours}}$.

A slope of −25 means the amount of water in the reservoir is decreasing (negative change) at a rate of 25 thousand cubic feet each hour.

CHECK IT OUT! **3.** The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.



$\frac{1}{2}$; the height of the plant is increasing at a rate of 1 cm every 2 days.

If you know the equation that describes a line, you can find its slope by using any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

EXAMPLE 4 Finding Slope from an Equation

Find the slope of the line described by $6x - 5y = 30$.

Step 1 Find the *x*-intercept.

$6x - 5y = 30$
$6x - 5(0) = 30$ *Let y = 0.*
$6x = 30$
$\frac{6x}{6} = \frac{30}{6}$
$x = 5$

Step 2 Find the *y*-intercept.

$6x - 5y = 30$
$6(0) - 5y = 30$ *Let x = 0.*
$-5y = 30$
$\frac{-5y}{-5} = \frac{30}{-5}$
$y = -6$

Step 3 The line contains $(5, 0)$ and $(0, -6)$. Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 5} = \frac{-6}{-5} = \frac{6}{5}$$

CHECK IT OUT! **4.** Find the slope of the line described by $2x + 3y = 12$. $-\frac{2}{3}$

3 Close

Summarize

Write the coordinates $A(9, 2)$ and $B(5, -1)$ on the board. Have half of the class use the slope formula with *A* as (x_1, y_1), and have the other half use *B* as (x_1, y_1). Compare answers. Each method will give a slope of $\frac{3}{4}$. Remind students that it does not matter which point you choose to be (x_1, y_1), but once the choice is made, you must subtract the *y*-coordinates and *x*-coordinates in the same order.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

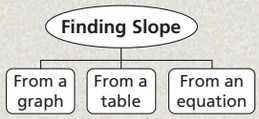
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. The slope of a line is the difference of the ____?____ divided by the difference of the ____?____ for any two points on the line.
2. Two points lie on a line. When you substitute their coordinates into the slope formula, the value of the denominator is 0. Describe this line.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find slope using the given method.

Know it! Note



4-4 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

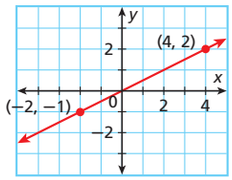
Find the slope of the line that contains each pair of points.

1. $(3, 6)$ and $(6, 9)$ 1
2. $(2, 7)$ and $(4, 4)$ $-\frac{3}{2}$
3. $(-1, -5)$ and $(-9, -1)$ $-\frac{1}{2}$

SEE EXAMPLE 2

Each graph or table shows a linear relationship. Find the slope.

4. $\frac{1}{2}$



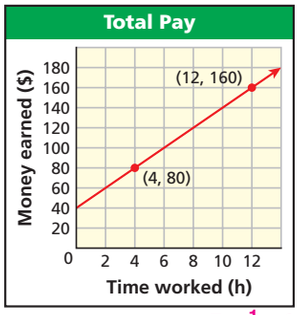
5. 10

x	y
0	25
2	45
4	65
6	85

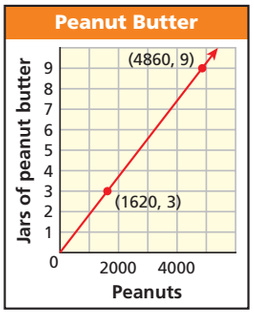
SEE EXAMPLE 3

Find the slope of each line. Then tell what the slope represents.

6. 10; the money earned is increasing at a rate of $10/h.



7. $\frac{1}{540}$; for each jar of peanut butter, 540 peanuts are needed.



SEE EXAMPLE 4

Find the slope of the line described by each equation.

8. $8x + 2y = 96$ -4
9. $5x = 90 - 9y$ $-\frac{5}{9}$
10. $5y = 160 + 9x$ $\frac{9}{5}$

Answers to *Think and Discuss*

1. *y*-values; *x*-values
2. vertical line
3. See Additional Answers.

4-4 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 11–15, 22
Average 11–15, 22, 29, 30
Advanced 11–15, 29–36

If you finished Examples **1–4**
Basic 11–22, 25–28
Average 11–28, 32–37
Advanced 11–20, 22–37

Homework Quick Check
Quickly check key concepts.
Exercises: 12, 14, 16, 18, 22

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 22, 25

Reason abstractly and quantitatively. Exercises 6–7, 16–17

Construct viable arguments and critique the reasoning of others. Exercises 21, 23–24

Look for and make use of structure. Exercises 1–5, 8–15, 18–20, 29–37

Learn It Online
State Resources Online

MULTI-STEP TEST PREP **Exercise 25** involves writing a function to describe a relationship, calculating the slope, and interpreting the slope in a real-world context. This exercise prepares students for the Multi-Step Test Prep.

Answers

23b. The speed and the slope are both equal to the dist. divided by time.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–13	1
14–15	2
16–17	3
18–20	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

Find the slope of the line that contains each pair of points.

11. (2, 5) and (3, 1) **−4** **12.** (−9, −5) and (6, −5) **0** **13.** (3, 4) and (3, −1) **undef.**

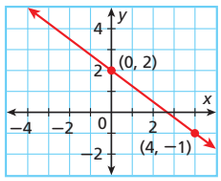
Each graph or table shows a linear relationship. Find the slope.

14.

x	y
1	18.5
2	22
3	25.5
4	29

$\frac{7}{2}$

15.



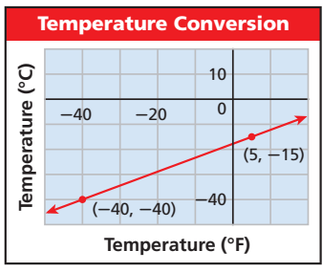
$-\frac{3}{4}$

Find the slope of each line. Then tell what the slope represents.

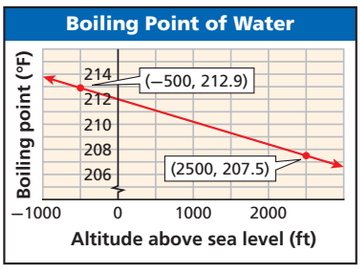
16. $\frac{5}{9}$; temp. in Celsius is increasing at a rate of 5 °C for each 9 °F.

17. $-\frac{9}{5000}$; the boiling point is decreasing at a rate of 9 °F for each 5000 ft above sea level.

16.



17.



Find the slope of the line described by each equation.

18. $7x + 13y = 91$ $-\frac{7}{13}$ **19.** $5y = 130 - 13x$ $-\frac{13}{5}$ **20.** $7 - 3y = 9x$ **−3**

21. ERROR ANALYSIS Two students found the slope of the line that contains $(-6, 3)$ and $(2, -1)$. Who is incorrect? Explain the error. **Student B. The student did not use the same coordinate pair order in the denom. as in the num.**

A

$$m = \frac{-1 - 3}{2 - (-6)} = \frac{-4}{8} = -\frac{1}{2}$$

B

$$m = \frac{-1 - 3}{-6 - 2} = \frac{-4}{-8} = \frac{1}{2}$$

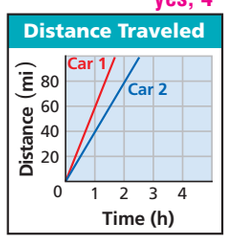
22. Environmental Science The table shows how the number of cricket chirps per minute changes with the air temperature.

Temperature (°F)	40	50	60	70	80	90
Chirps per minute	0	40	80	120	160	200

a. Find the rates of change. **The rate of change for each interval is $\frac{4 \text{ chirps/min}}{1 °F}$.**

b. Is the graph of the data a line? If so, what is the slope? If not, explain why not. **yes; 4**

23. Critical Thinking The graph shows the distance traveled by two cars.



a. Which car is going faster? How much faster? **car 1; 20 mi/h**

b. How are the speeds related to slope?

c. At what rate is the distance between the cars changing? **20 mi/h**

24. Write About It You are given the coordinates of two points on a line. Describe two different ways to find the slope of that line.

24. Possible answer: Given the 2 pts. (x_1, y_1) and (x_2, y_2), you could substitute into the slope formula or graph the 2 points, connect with a line, and count the rise and run.

PRACTICE A

PRACTICE C

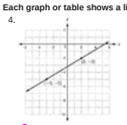
PRACTICE B

Find the slope of the line that contains each pair of points.

1. (2, 8) and (1, −3) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 8}{1 - 2} = \frac{-11}{-1} = 11$
2. (−4, 0) and (−6, −2) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{-6 - -4} = \frac{-2}{-2} = 1$
3. (0, −2) and (4, −7) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - -2}{4 - 0} = \frac{-5}{4}$

Each graph or table shows a linear relationship. Find the slope.

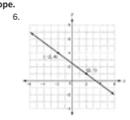
4.



5.

x	y
1	3.75
2	5
3	6.25
4	7.50
5	8.75

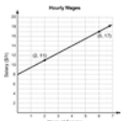
6.



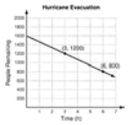
$\frac{3}{5}$ 1.25 $-\frac{3}{4}$

Find the slope of each line. Then tell what the slope represents.

7.



8.



$\frac{3}{2}$; the salary increases $3 every 2 years. $-\frac{400}{3}$; the number of people remaining decreases by 400 every 3 hours.

Find the slope of the line described by each equation.

9. $3x + 4y = 24$ $-\frac{3}{4}$
10. $8x + 48 = 3y$ $\frac{8}{3}$

READING STRATEGIES

In the slope formula, x- and y-values of two different points are identified using subscripts, as described below.

Read: "y sub 2" **Mean:** y-value of the 2nd point

Read: "y sub 1" **Mean:** y-value of the 1st point

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Read: "x sub 2" **Mean:** x-value of the 2nd point

Read: "x sub 1" **Mean:** x-value of the 1st point

Answer questions 1–3 using the table below.

x	−3	−2	−1	0	1
y	2	4	6	8	10

1. If you use −2 to represent x_1, what is the value of y_1? 4
2. Choose a different point to represent (x_2, y_2). **Possible answer: (−1, 6)**
3. Calculate the slope of the line represented by the table. 2

Use the graph to complete questions 4 and 5.

4. Which two ordered pairs are plotted on the graph? **(−4, 1) and (6, 5)**
5. What is the slope of the line? $\frac{2}{5}$
6. Find the slope of the line that contains (3, 6) and (−4, 6). 0
7. Describe the line in problem 6. **horizontal**

RETEACH

You can find the slope of a line from any two ordered pairs. The ordered pairs can be given to you, or you might need to read them from a table or graph.

Find the slope of the line that contains (−1, 3) and (2, 0).

Step 1: Name the ordered pairs. (It does not matter which is first and which is second.)

first ordered pair (−1, 3) (2, 0) second ordered pair

Step 2: Label each number in the ordered pairs.

$(-1, 3)$ $(2, 0)$
(x_1, y_1) (x_2, y_2)

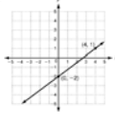
Step 3: Substitute the ordered pairs into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{2 - (-1)} = \frac{-3}{3} = -1$$

The slope of the line that contains (−1, 3) and (2, 0) is −1.

Find the slope of each linear relationship.

1.



2.

x	y
4	−5
8	−3
12	−1
16	1

3. The line contains (5, −2) and (7, 6).

$\frac{3}{4}$ $\frac{1}{2}$ 4

MULTI-STEP TEST PREP

25. a. One way to estimate your maximum heart rate is to subtract your age from 220. Write a function to describe the relationship between maximum heart rate y and age x. $y = 220 - x$

b. The graph of this function is a line. Find its slope. Then tell what the slope represents.

TEST PREP

26. The equation $2y + 3x = -6$ describes a line with what slope?

Ⓐ $\frac{3}{2}$ Ⓑ 0 Ⓒ $\frac{1}{2}$ Ⓓ $-\frac{3}{2}$

27. A line with slope $-\frac{1}{3}$ could pass through which of the following pairs of points?

Ⓕ $\left(0, -\frac{1}{3}\right)$ and $(1, 1)$ Ⓗ $(0, 0)$ and $\left(-\frac{1}{3}, -\frac{1}{3}\right)$

Ⓖ $(-6, 5)$ and $(-3, 4)$ Ⓙ $(5, -6)$ and $(4, 3)$

28. Gridded Response Find the slope of the line that contains $(-1, 2)$ and $(5, 5)$. $\frac{1}{2}$, or 0.5

CHALLENGE AND EXTEND

Find the slope of the line that contains each pair of points.

29. $(a, 0)$ and $(0, b)$ $-\frac{b}{a}$ **30.** $(2x, y)$ and $(x, 3y)$ $-\frac{2y}{x}$ **31.** (x, y) and $(x + 2, 3 - y)$ $\frac{3}{2} - y$

Find the value of x so that the points lie on a line with the given slope.

32. $(x, 2)$ and $(-5, 8)$, $m = -1$ 1 **33.** $(4, x)$ and $(6, 3x)$, $m = \frac{1}{2}$ $\frac{1}{2}$

34. $(1, -3)$ and $(3, x)$, $m = -1$ −5 **35.** $(-10, -4)$ and (x, x), $m = \frac{1}{7}$ −3

36. A line contains the point $(1, 2)$ and has a slope of $\frac{1}{2}$. Use the slope formula to find another point on this line. Possible answer: $(3, 3)$

37. The points $(-2, 4)$, $(0, 2)$, and $(3, x - 1)$ all lie on the same line. What is the value of x? (*Hint:* Remember that the slope of a line is constant for any two points on the line.) 0

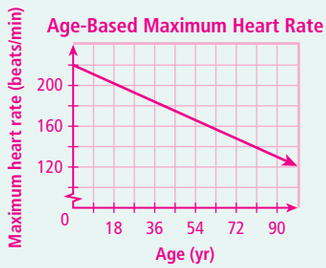
Victoria Smith/HMH

TEST PREP DOCTOR

In **Exercise 26,** students who chose **A** may have subtracted the coordinates in different orders or made an error in integer subtraction.

Answers

25b.



The slope is −1; for every year increase, the max. heart rate decreases 1 beat/min.

Journal

Have students describe similarities and differences between each method of finding slope—counting the rise and run and using the formula.

ALTERNATIVE ASSESSMENT

Have students graph four lines with different slopes—one positive, one negative, one zero, and one undefined. They should then label two points on each and find each slope by using the slope formula.

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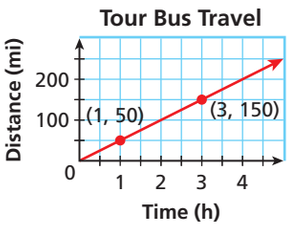
4-4 Lesson Quiz

1. Find the slope of the line that contains $(5, 3)$ and $(-1, 4)$. $-\frac{1}{6}$

2. Find the slope of the line. Then tell what the slope represents.



50; speed of bus is 50 mi/h

3. Find the slope of the line described by $x + 2y = 8$. $-\frac{1}{2}$

Also available online

PROBLEM SOLVING

Write the correct answer.

1. The graph shows the number of emergency kits assembled by volunteers over a period of days. Find the slope of the line. Then tell what the slope represents.

2. The graph shows how much flour is in a bag at different times. Find the slope of the line. Then tell what the slope represents.

12; the number of kits assembled per day.

0.2; the number of pounds of flour used per day.

3. The function $20x - y = 250$ describes the profit y Bridget can make from selling x pairs of earrings. The graph of this function is a line. Find its slope. 20

The graph below shows the cost of membership at Fabulously Fit. Use the graph to answer questions 4–7. Select the best answer.

4. What is the slope of the line?
   A 24 C 50
   B 35 D 70

5. What does the slope represent?
   F the enrollment fee
   G the late payment fee
   H the total cost of membership
   J the monthly membership fee

6. A second line is graphed that shows the cost of membership at The Fitness Studio. The line contains (0, 35) and (5, 85). What is the slope of this line?
   A 10 C 45
   B 20 D 50

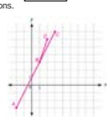
7. How much greater is the monthly fee at Fabulously Fit than The Fitness Studio?
   F $15 H $35
   G $25 J $40

CHALLENGE

When two variables x and y are related and (x_1, y_1) and (x_2, y_2) satisfy the relationship, you can write the quotient as shown at right. The quotient is called a *difference quotient.* $\frac{y_2 - y_1}{x_2 - x_1}$

The difference quotient can have different meanings in different situations.

In Exercises 1–5, use the grid at the right.



1. Graph and connect $A(-2, -3)$, $B(1, 3)$, and $C(3, 7)$.
2. Calculate the slope of $\overline{AB}$ and of $\overline{BC}$. Slope of $\overline{AB}$ = Slope of $\overline{BC}$ = 2
3. On the same grid, graph $D(2, 6)$. Connect A, B and D.
4. Calculate the slope of $\overline{AB}$ and of $\overline{BD}$. Slope of $\overline{AB}$ = 2; Slope of $\overline{BD}$ = 3
5. Explain how to use a difference quotient to determine whether three points are collinear, that is, whether three points lie on the same line.

Points A, B, and C lie on a line if the slopes of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are equal.

Another application of the difference quotient is the measurement of the *grade,* or steepness, of a road. A grade of 3% means that the road surface rises 3 feet for every horizontal run of 100 feet. A roadway that slopes down has a negative grade.

6. If the vertical rise of a highway is 150 feet when the horizontal run is 5000 feet, what is the grade of the road? 3%
7. If the grade of a road is 5%, what is the vertical rise of the road when the horizontal run is 6000 feet? 300 feet
8. On a certain stretch of road, the point 420 feet east of point A is 24 feet lower than point A. Point B, 600 feet east of A, is 36 feet lower than point A.
   a. Find the grade of the road surface between points A and B. 6%
   b. Would a motorist driving along a stretch of road from point B to point A report the same grade for the road as a motorist driving from A to B would report? Explain your response.

No; the uphill grade is positive and the downhill grade is negative.

In economics, the additional cost to produce one more unit of an item is called the *marginal cost* of the item. Marginal cost is found by computing the difference quotient, or difference in $\frac{\text{manufacturing cost}}{\text{difference in units made}}$.

9. The cost that a manufacturer pays for producing 5 pairs of shoes is $113. The cost to produce 12 pairs of shoes is $127. Use a difference quotient to find the marginal cost. $2

4-5 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Identify, write, and graph direct variation.

Online Edition
Tutorial Videos

Countdown

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Warm Up

Solve for *y*.

1. $3 + y = 2x$ $y = 2x - 3$

2. $6x = 3y$ $y = 2x$

Write an equation that describes the relationship.

3.

x	1	2	3	4
y	3	6	9	12

$y = 3x$

Solve for *x*.

4. $\frac{3}{5} = \frac{x}{15}$ 9

5. $\frac{6}{2} = \frac{1.5}{x}$ 0.5

Also available online

Math Humor

Parent: How is your math homework going?

Child: Well, *m* is negative, so at least I'm on the down slope!

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

CC.9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.* b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

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COMMON CORE

4-5 Direct Variation

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *Also* **CC.9-12.F.LE.1*, CC.9-12.F.LE.2*, CC.9-12.A.CED.3*, CC.9-12.F.IF.5*, CC.9-12.F.IF.7***

Objective
Identify, write, and graph direct variation.

Vocabulary
direct variation
constant of variation

Who uses this?
Chefs can use direct variation to determine ingredients needed for a certain number of servings.

Paella is a rice dish that originated in Valencia, Spain.

A recipe for paella calls for 1 cup of rice to make 5 servings. In other words, a chef needs 1 cup of rice for every 5 servings.

Rice (c) *x*	1	2	3	4
Servings *y*	5	10	15	20

The equation $y = 5x$ describes this relationship. In this relationship, the number of servings *varies directly* with the number of cups of rice.

A **direct variation** is a special type of linear relationship that can be written in the form $y = kx$, where k is a nonzero constant called the **constant of variation**.

EXAMPLE 1 Identifying Direct Variations from Equations

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

A $y = 4x$

This equation represents a direct variation because it is in the form $y = kx$. The constant of variation is 4.

B $-3x + 5y = 0$

$-3x + 5y = 0$ *Solve the equation for y.*

$+3x \quad +3x$ *Since −3x is added to 5y, add 3x to both sides.*

$5y = 3x$

$\frac{5y}{5} = \frac{3x}{5}$ *Since y is multiplied by 5, divide both sides by 5.*

$y = \frac{3}{5}x$

This equation represents a direct variation because it can be written in the form $y = kx$. The constant of variation is $\frac{3}{5}$.

C $2x + y = 10$

$2x + y = 10$ *Solve the equation for y.*

$-2x \quad -2x$ *Since 2x is added to y, subtract 2x from both sides.*

$y = -2x + 10$

This equation does not represent a direct variation because it cannot be written in the form $y = kx$.

CHECK IT OUT! **Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

1a. $3y = 4x + 1$ **1b.** $3x = -4y$ **1c.** $y + 3x = 0$

1a. no
1b. yes; $-\frac{3}{4}$
1c. yes; −3

© Steve Lupton/Corbis

1 Introduce

EXPLORATION

Direct Variation

You will need a graphing calculator for this Exploration.

1. Use your calculator to graph the following functions in the same viewing window.
$y = 3x$
$y = \frac{1}{2}x$
$y = 5x$

2. What do you notice about the graphs? What do they all have in common? All are straight lines through (0, 0).
3. Make a table for the function $y = 3x$ as follows. Press [Y=] and delete the functions that have been entered for **Y2** and **Y3**. Then press [2nd] [GRAPH] (TABLE) to view the table for **Y1**.
4. Choose any row of the table except the row for $x = 0$. Divide the y-value by the x-value. What is the result? Is this true for every row of the table? 3; yes
5. Follow the same procedure to examine the tables for $y = \frac{1}{2}x$ and $y = 5x$. For each row (except $x = 0$), divide the y-value by the x-value. Describe the results for each function.

THINK AND DISCUSS

6. **Predict** what would happen if you made a table of values for $y = 7x$ and divided any y-value by its corresponding x-value (except $x = 0$). The quotient will always be 7.
7. **Describe** the graph of an equation that has the form $y = kx$,

Motivate

Have a volunteer count the number of people present in the classroom and multiply the number by 2. Ask what the product could represent in relation to the number of people in the room. Possible answer: number of shoes What could the product represent if you multiplied by 10? Possible answer: number of fingers

Tell students that these are examples of direct variations.

Explorations and answers are provided in your online resources.

What happens if you solve $y = kx$ for k?

$y = kx$

$\frac{y}{x} = \frac{kx}{x}$ *Divide both sides by x ($x \neq 0$).*

$\frac{y}{x} = k$

So, in a direct variation, the ratio $\frac{y}{x}$ is equal to the constant of variation. Another way to identify a direct variation is to check whether $\frac{y}{x}$ is the same for each ordered pair (except where $x = 0$).

EXAMPLE 2 Identifying Direct Variations from Ordered Pairs

Tell whether each relationship is a direct variation. Explain.

A

x	1	3	5
y	6	18	30

Method 1 Write an equation.

$y = 6x$ *Each y-value is 6 times the corresponding x-value.*

This is a direct variation because it can be written as $y = kx$, where $k = 6$.

Method 2 Find $\frac{y}{x}$ for each ordered pair.

$\frac{6}{1} = 6$ $\frac{18}{3} = 6$ $\frac{30}{5} = 6$

This is a direct variation because $\frac{y}{x}$ is the same for each ordered pair.

B

x	2	4	8
y	−2	0	4

Method 1 Write an equation.

$y = x - 4$ *Each y-value is 4 less than the corresponding x-value.*

This is not a direct variation because it cannot be written as $y = kx$.

Method 2 Find $\frac{y}{x}$ for each ordered pair.

$\frac{-2}{2} = -1$ $\frac{0}{4} = 0$ $\frac{4}{8} = \frac{1}{2}$

This is not a direct variation because $\frac{y}{x}$ is not the same for all ordered pairs.

CHECK IT OUT! **Tell whether each relationship is a direct variation. Explain.**

2a.

x	y
−3	0
1	3
3	6

2b.

x	y
2.5	−10
5	−20
7.5	−30

2c.

x	y
−2	5
1	3
4	1

2a. No; possible answer: the value of $\frac{y}{x}$ is not the same for each ordered pair.

2b. Yes; possible answer: the value of $\frac{y}{x}$ is the same for each ordered pair.

2c. No; possible answer: the value of $\frac{y}{x}$ is not the same for each ordered pair.

If you know one ordered pair that satisfies a direct variation, you can write the equation. You can also find other ordered pairs that satisfy the direct variation.

COMMON ERROR ALERT

Students may have trouble differentiating between a direct variation and a simple linear relationship like $y = x + 2$. Explain that although both $y = 2x$ and $y = x + 2$ are linear functions, the relationship between x and y in a direct variation always has a y-intercept of 0.

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Additional Examples

Example 1

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

A. $y = 3x$ yes; 3

B. $3x + y = 8$ no

C. $-4x + 3y = 0$ yes; $\frac{4}{3}$

Example 2

Tell whether each relationship is a direct variation. Explain.

A.

x	2	4	6
y	6	12	18

Yes; possible answer: the value of $\frac{y}{x}$ is the same for each ordered pair.

B.

x	1	3	7
y	−2	0	4

No; possible answer: the value of $\frac{y}{x}$ is not the same for each ordered pair.

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- Is the constant of variation always the coefficient of x? Explain.

EXAMPLE 2

- Why does finding $\frac{y}{x}$ for each ordered pair tell you whether the relationship is a direct variation?
- If the coefficient of x is 1, can the relationship be a direct variation? Explain.

Teaching Tip **Inclusion** Point out that a linear equation cannot be a direct variation if the value of C is nonzero when the equation is written in standard form ($Ax + By = C$).

2 Teach

Guided Instruction

Review writing an equation from a table. Remind students that they are looking for a consistent relationship between x and y in which y is always a constant multiple of x. Before **Example 3,** review solving a proportion by using cross products.

Teaching Tip **Visual** Some students may find a table of values easier to read if the values are written in columns rather than rows.

Reaching All Learners

Through Multiple Representations

Have students work in small groups. Using a ruler (MK), each group should measure and record the thickness of one algebra book. Then have them measure and record the height of 2 algebra books, 3 algebra books, and so on, until all group members' books are on the stack. Using these data, ask each group to explain how the relationship between the number of books and the height of the stack models a direct variation.

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Additional Examples

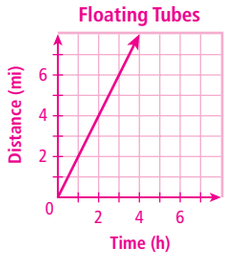
Example 3

The value of y varies directly with x, and $y = 3$ when $x = 9$. Find y when $x = 21$. 7

Example 4

A group of people are tubing down a river at an average speed of 2 mi/h. Write a direct variation equation that gives the number of miles y that the people will float in x hours. Then graph.

$y = 2x$



Also available online

Questioning Strategies

EXAMPLE 3

- If you use a proportion to find y, how can you then find k?

EXAMPLE 4

- How can you "see" the constant of variation in the ordered pairs? in the graph?

Teaching Tip **Critical Thinking** Ask students whether the graph in **Example 4** appears to suggest that the three-toed sloth is *not* "an extremely slow animal." Point out that the steep slope represents a rate of 6 ft/min, which is equivalent to about 0.07 mi/h.

EXAMPLE 3 Writing and Solving Direct Variation Equations

The value of y varies directly with x, and $y = 6$ when $x = 12$. Find y when $x = 27$.

Method 1 Find the value of k and then write the equation.

$y = kx$ *Write the equation for a direct variation.*

$6 = k(12)$ *Substitute 6 for y and 12 for x. Solve for k.*

$\frac{1}{2} = k$ *Since k is multiplied by 12, divide both sides by 12.*

The equation is $y = \frac{1}{2}x$. When $x = 27$, $y = \frac{1}{2}(27) = 13.5$.

Method 2 Use a proportion.

$\frac{6}{12} = \frac{y}{27}$ *In a direct variation, $\frac{y}{x}$ is the same for all values of x and y.*

$12y = 162$ *Use cross products.*

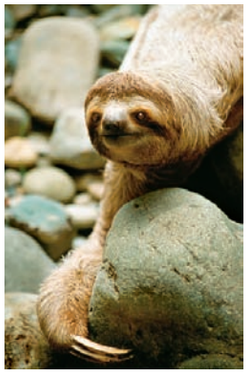
$y = 13.5$ *Since y is multiplied by 12, divide both sides by 12.*

CHECK IT OUT!

3. The value of y varies directly with x, and $y = 4.5$ when $x = 0.5$. Find y when $x = 10$. **90**

EXAMPLE 4 Graphing Direct Variations

The three-toed sloth is an extremely slow animal. On the ground, it travels at a speed of about 6 feet per minute. Write a direct variation equation for the distance y a sloth will travel in x minutes. Then graph.



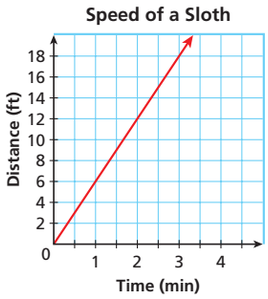
Step 1 Write a direct variation equation.

distance	=	6 feet per minute	times	number of minutes
y	=	6	·	x

Step 2 Choose values of x and generate ordered pairs.

x	$y = 6x$	(x, y)
0	$y = 6(0) = 0$	(0, 0)
1	$y = 6(1) = 6$	(1, 6)
2	$y = 6(2) = 12$	(2, 12)

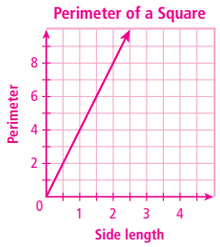
Step 3 Graph the points and connect.



CHECK IT OUT!

4. The perimeter y of a square varies directly with its side length x. Write a direct variation equation for this relationship. Then graph.

4. $y = 4x$



Look at the graph in Example 4. It passes through $(0, 0)$ and has a slope of 6. The graph of any direct variation $y = kx$

- is a line through $(0, 0)$.
- has a slope of k.

Geneviève Vallée/Alamy

3 Close

Summarize

Ask students if each relationship or equation below is a direct variation. If so, have them identify the constant of variation.

x	0	3	6
y	2	8	14

no; $\frac{2}{0} \neq \frac{8}{3} \neq \frac{14}{6}$

$x - 4y = 0$ yes; $\frac{1}{4}$

$5x + 10 = y$ no; cannot be written in the form $y = kx$.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. How do you know that a direct variation is linear?
2. How does the graph of a direct variation differ from the graphs of other types of linear relationships?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how you can use the given information to identify a direct variation.

Recognizing a Direct Variation		
From an Equation	From Ordered Pairs	From a Graph

Answers to *Think and Discuss*

Possible answers:

1. It can be written in standard form $kx - y = 0$ with $A = k$, $B = -1$, and $C = 0$.
2. The graph of a direct variation always passes through the origin and is neither horizontal nor vertical.
3. See Additional Answers.

4-5 Exercises

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4-5 Exercises

GUIDED PRACTICE

1. **Vocabulary** If x varies directly with y, then the relationship between the two variables is said to be a ___?___. (*direct variation* or *constant of variation*) direct variation

SEE EXAMPLE 1

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

2. $y = 4x + 9$ no
3. $2y = -8x$ yes; -4
4. $x + y = 0$ yes; -1

SEE EXAMPLE 2

Tell whether each relationship is a direct variation. Explain.

5. no

x	10	5	2
y	12	7	4

6. yes

x	3	-1	-4
y	-6	2	8

SEE EXAMPLE 3

7. The value of y varies directly with x, and $y = -3$ when $x = 1$. Find y when $x = -6$. 18
8. The value of y varies directly with x, and $y = 6$ when $x = 18$. Find y when $x = 12$. 4

SEE EXAMPLE 4

9. **Wages** Cameron earns \$7 per hour at her after-school job. The total amount of her paycheck varies directly with the amount of time she works. Write a direct variation equation for the amount of money y that she earns for working x hours. Then graph. $y = 7x$

PRACTICE AND PROBLEM SOLVING

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

10. $y = \frac{1}{6}x$ yes; $\frac{1}{6}$
11. $4y = x$ yes; $\frac{1}{4}$
12. $x = 2y - 12$ no

Tell whether each relationship is a direct variation. Explain.

13. yes

x	6	9	17
y	13.2	19.8	37.4

14. yes

x	-6	3	12
y	4	-2	-8

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–2
Basic 10–14, 18, 19
Average 10–14, 18, 19
Advanced 10–14, 18, 19, 46

If you finished Examples 1–4
Basic 10–37, 40–44
Average 10–17, 20–44, 46
Advanced 10–17, 20–46

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 14, 16, 17, 22, 36

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 9, 17, 20–37, 40, 45

Construct viable arguments and critique the reasoning of others. Exercises 18–19, 38–39

Look for and make use of structure. Exercises 2–8, 10–16, 46

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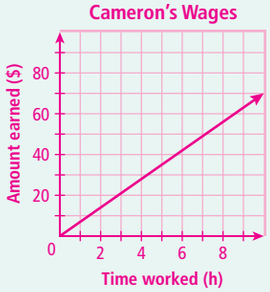
Answers

5. Possible answer: The value of $\frac{y}{x}$ is not the same for each ordered pair.
6. Possible answer: The value of $\frac{y}{x}$ is the same for each ordered pair.
9.

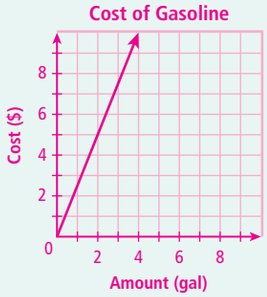


13. Possible answer: The value of $\frac{y}{x}$ is the same for each ordered pair.
14. Possible answer: The value of $\frac{y}{x}$ is the same for each ordered pair.

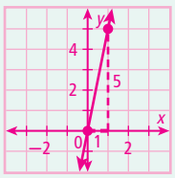
MULTI-STEP TEST PREP **Exercise 40** involves writing an equation of direct variation and determining the constant of variation. This exercise prepares students for the Multi-Step Test Prep.

Answers

17.

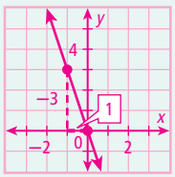


20. $y = 5x$



$k = 5$; graph shows slope = 5

21. $y = -3x$



$k = -3$; graph shows slope = -3

22–35, 37b, 38. See Additional Answers.

39. Possible answer: The ratio $\frac{y}{x}$ is the same for all ordered pairs in a direct var., so you can write a proportion using any two ordered pairs.

Independent Practice

For Exercises	See Example
10–12	1
13–14	2
15–16	3
17	4

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

15. The value of y varies directly with x, and $y = 8$ when $x = -32$. Find y when $x = 64$. **−16**

16. The value of y varies directly with x, and $y = \frac{1}{2}$ when $x = 3$. Find y when $x = 1$. **$\frac{1}{6}$**

17. While on his way to school, Norman saw that the cost of gasoline was $2.50 per gallon. Write a direct variation equation to describe the cost y of x gallons of gas. Then graph. **$y = 2.50x$**

Tell whether each relationship is a direct variation. Explain your answer.

18. The equation $-15x + 4y = 0$ relates the length of a videotape in inches x to its approximate playing time in seconds y. **Yes; it can be written as $y = \frac{15}{4}x$.**

19. The equation $y - 2.00x = 2.50$ relates the cost y of a taxicab ride to distance x of the cab ride in miles. **No; it cannot be written in the form $y = kx$.**

Each ordered pair is a solution of a direct variation. Write the equation of direct variation. Then graph your equation and show that the slope of the line is equal to the constant of variation.

20. $(2, 10)$ **21.** $(-3, 9)$ **22.** $(8, 2)$ **23.** $(1.5, 6)$

24. $(7, 21)$ **25.** $(1, 2)$ **26.** $(2, -16)$ **27.** $\left(\frac{1}{7}, 1\right)$

28. $(-2, 9)$ **29.** $(9, -2)$ **30.** $(4, 6)$ **31.** $(3, 4)$

32. $(5, 1)$ **33.** $(1, -6)$ **34.** $\left(-1, \frac{1}{2}\right)$ **35.** $(7, 2)$

LINK Astronomy

The Mars rover *Spirit* landed on Mars in January 2004 and immediately began sending photos of the planet's surface back to Earth.

36. Astronomy Weight varies directly with gravity. A Mars lander weighed 767 pounds on Earth but only 291 pounds on Mars. Its accompanying Mars rover weighed 155 pounds on Mars. How much did it weigh on Earth? Round your answer to the nearest pound. **409 lb**

37. Environment Mischa bought an energy-efficient washing machine. She will save about 15 gallons of water per wash load.

a. Write an equation of direct variation to describe how many gallons of water y Mischa saves for x loads of laundry she washes. **$y = 15x$**

b. Graph your direct variation from part **a.** Is every point on the graph a solution in this situation? Why or why not?

c. If Mischa does 2 loads of laundry per week, how many gallons of water will she have saved at the end of a year? **1560 gal**

38. Critical Thinking If you double an x-value in a direct variation, will the corresponding y-value double? Explain.

39. Write About It In a direct variation $y = kx$, k is sometimes called the "constant of proportionality." How are proportions related to direct variations?

MULTI-STEP TEST PREP

40. Rhea exercised on a treadmill at the gym. When she was finished, the display showed that she had walked at an average speed of 3 miles per hour.

a. Write an equation that gives the number of miles y that Rhea would cover in x hours if she walked at this speed. **$y = 3x$**

b. Explain why this is a direct variation and find the value of k. What does this value represent in Rhea's situation? **It is written in the form $y = kx$, where $k = 3$. This value represents the speed at which Rhea is walking.**

(cl),Courtesy NASA/JPL-Caltech; (bl),Victoria Smith/HMH

PRACTICE A
PRACTICE C
PRACTICE B

Tell whether each equation is a direct variation. If so, identify the constant of variation.

1. $y = 3x$ **yes; 3**
2. $y = 2x - 9$ **no**
3. $2x + 3y = 0$ **yes; $-\frac{2}{3}$**
4. $3y = 9x$ **yes; 3**

Find the value of $\frac{y}{x}$ for each ordered pair. Then, tell whether each relationship is a direct variation.

5.

x	6	15	21
y	2	5	7
$\frac{y}{x}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

yes

6.

x	6	10	25
y	24	40	100
$\frac{y}{x}$	4	4	4

yes

7.

x	10	15	20
y	3	5	9
$\frac{y}{x}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{9}{20}$

no

8. The value of y varies directly with x, and $y = -18$ when $x = 6$. Find y when $x = -8$.

Find k: $-3 = k$

Use k to find y: $y = kx$; $y = (-3)(-8)$; $y = 24$

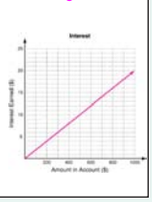
Find k: $\frac{1}{10} = k$

Use k to find y: $y = kx$; $y = \left(\frac{1}{10}\right)(30)$; $y = 3$

9. The value of y varies directly with x, and $y = \frac{1}{2}$ when $x = 5$. Find y when $x = 30$.

10. The amount of interest earned in a savings account varies directly with the amount of money in the account. A certain bank offers a 2% savings rate. Write a direct variation equation for the amount of interest y earned on a balance of x. Then graph.

$y = 0.02x$



11. Another bank offers a different savings rate. If an account with $400 earns interest of $6, how much interest is earned by an account with $1800?

$27

READING STRATEGIES

A *direct variation* is a special type of linear relationship. It can be expressed using the formula $y = kx$, where k is the *constant of variation*.

The equation $y = \frac{3}{4}x$ is a direct variation because it is in the form $y = kx$, where $k = \frac{3}{4}$.

x	2	3	4
y	−4	−6	−8

The relationship shown in the table is a direct variation because $k = \frac{y}{x} = -2$ for each ordered pair.

$y = kx$

$k = \frac{y}{x}$

The value of y varies directly with x and $y = 5$ when $x = 2$.

$y = kx$
$5 = k2$
$\frac{5}{2} = k$

Find y when $x = 8$.
$y = \frac{5}{2}(8)$
$y = 20$

Identify k, the constant of variation, for each direct variation.

1. $y = 8x$ **8**
2. $y = \frac{x}{3}$ **$\frac{1}{3}$**
3. $-2y = 5x$ **−2.5**

Solve each equation for y. Tell whether or not it is a direct variation.

4. $x + y = 12$ **$y = 12 - x$; no**
5. $6x = 4y$ **$y = \frac{3}{2}x$; yes**
6. Calculate $\frac{y}{x}$ for each ordered pair in the table.

x	−4	−2	4
y	−2	−1	2

$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

7. Is this a direct variation? **yes**
8. The value of y varies directly with x, and $y = 21$ when $x = 7$. Find y when $x = 4$. **12**

RETEACH

A direct variation is a special type of linear relationship. It can be written in the form $y = kx$ where k is a nonzero constant called the constant of variation.

You can identify direct variations from equations or from ordered pairs.

Tell whether the relationship is a direct variation. If so, identify the constant of variation.

First, put the equation in the form $y = kx$.

$2x + 4y = 0$
$-2x \quad -2x$ — Add $-2x$ to each side.
$4y = -2x$
$\frac{4y}{4} = -\frac{2x}{4}$ — Divide both sides by 4.
$y = -\frac{1}{2}x$

Because the equation can be written in the form $y = kx$, it is a direct variation.

The constant of variation is $-\frac{1}{2}$.

Tell whether the relationship is a direct variation. If so, identify the constant of variation.

x	2	4	6
y	1	2	3

If we solve $y = kx$ for k, we get:

$y = kx \longrightarrow \frac{y}{x} = \frac{kx}{x} \longrightarrow \frac{y}{x} = k$

Find k for each ordered pair. This means find $\frac{y}{x}$ for each ordered pair. If they are the same, the relationship is a direct variation.

$\frac{1}{2} \quad \frac{2}{4} = \frac{1}{2} \quad \frac{3}{6} = \frac{1}{2}$

This is a direct variation.

Tell whether each equation or relationship is a direct variation. If so, identify the constant of variation.

1. $x + y = 7$ **no**
2. $4x - 3y = 0$ **yes; $\frac{4}{3}$**
3. $-8y = 24x$ **yes; −3**

4.

x	−4	2	10
y	2	−1	−5

yes; $-\frac{1}{2}$

5.

x	5	12	8
y	17.5	42	28

yes; 3.5

6.

x	6	8	10
y	8	10	12

no

TEST PREP

41. Which equation does NOT represent a direct variation?

Ⓐ $y = \frac{1}{3}x$ Ⓑ $y = -2x$ Ⓒ $y = 4x + 1$ Ⓓ $6x - y = 0$

42. Identify which set of data represents a direct variation.

Ⓕ

x	1	2	3
y	1	2	3

Ⓗ

x	1	2	3
y	3	5	7

Ⓖ

x	1	2	3
y	0	1	2

Ⓙ

x	1	2	3
y	3	4	5

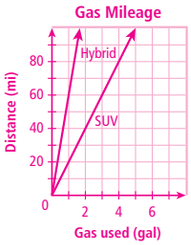
43. Two yards of fabric cost \$13, and 5 yards of fabric cost \$32.50. Which equation relates the cost of the fabric c to its length ℓ?

Ⓐ $c = 2.6\ell$ Ⓑ $c = 6.5\ell$ Ⓒ $c = 13\ell$ Ⓓ $c = 32.5\ell$

44. Gridded Response A car is traveling at a constant speed. After 3 hours, the car has traveled 180 miles. If the car continues to travel at the same constant speed, how many hours will it take to travel a total of 270 miles? **4.5**

CHALLENGE AND EXTEND

45b.



45. Transportation The function $y = 20x$ gives the number of miles y that a sport-utility vehicle (SUV) can travel on x gallons of gas. The function $y = 60x$ gives the number of miles y that a hybrid car can travel on x gallons of gas.

a. If you drive 120 miles, how much gas will you save by driving the hybrid instead of the SUV? **4 gal**

b. Graph both functions on the same coordinate plane. Will the lines ever meet other than at the origin? Explain. **No; the lines begin at (0, 0) and then move apart.**

c. What if...? Shannon drives 15,000 miles in one year. How many gallons of gas will she use if she drives the SUV? the hybrid? **750 gal; 250 gal**

46. Suppose the equation $ax + by = c$, where a, b, and c are real numbers, describes a direct variation. What do you know about the value of c? **$c = 0$**

PROBLEM SOLVING

Write the correct answer.

1. Wesley earns \$6.50 per hour at the bookstore. The total amount of his paycheck varies directly with the number of hours he works. Write a direct variation equation for the amount of money y Wesley earns for working x hours.
$y = 6.5x$

2. The equation $-4x + y = 0$ relates the number of pages in a photo album y to the number of pictures in the album x. Tell whether the relationship is a direct variation. Explain your answer.
yes; it can be written as $y = 4x$.

3. The formula $9x - 5y = -160$ relates the temperature in degrees Fahrenheit y to the temperature in degrees Celsius x. Tell whether the relationship is a direct variation. Explain your answer.
no; it cannot be written in the form $y = kx$

4. The number of miles driven varies directly with the number of gallons of gas used. Erin drove 297 miles on 9 gallons of gas. How far would she be able to drive on 14 gallons of gas?
462 miles

Select the best answer.

5. The table shows the relationship between the number of lemons purchased and their cost.

Lemons x	1	2	3	4
Cost y	0.1	0.2	0.3	0.4

Is the relationship a direct variation?
A Yes; it can be written as $y = 0.1x$.
B Yes; it can be written as $y = 10x$.
C No; it cannot be written as $y = kx$.
D No; the relationship is not a function.

6. The table shows the relationship between the hours since sunrise and the temperature in degrees Celsius.

Hour x	1	2	3	4
Temp. y	25	26	28	32

Is the relationship a direct variation?
F Yes; it can be written as $y = 25x$.
G Yes; it can be written as $y = 8x$.
H No; it cannot be written as $y = kx$.
J No; the relationship is not a function.

7. The Diaz family is driving at a constant speed on the highway so their distance varies directly with their speed. They traveled 17.5 miles in 15 minutes. How far did they travel in 2 hours?
A 50 miles **C** 140 miles
B 70 miles D 262.5 miles (17.5 times 15)

8. On July 26, 2005, it rained a record 37 inches in Mumbai, India, in a 24-hour period. Which equation of direct variation relates the number of inches of rain y to the number of hours x?
F $y = \frac{24}{37}x$ H $y = 24x$
G $y = \frac{37}{24}x$ J $y = 37x$

CHALLENGE

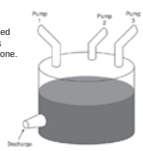
In Lesson 5.3, you learned that when y varies directly as x, there is a nonzero number, k, such that $y = kx$. There are many situations in which one variable varies directly with another variable.

A 700-gallon tank contains 500 gallons of a certain liquid. Three pumps feed three different ingredients into the tank at its top. A discharge pump at the base of the tank removes the mixed ingredients. The rates at which the pipes feed and remove liquid are shown at right.

Pump 1:	5 gallons per minute
Pump 2:	10 gallons per minute
Pump 3:	15 gallons per minute
Discharge pump:	35 gallons per minute

Let t represent the amount of time for which each pipe is operating.

1. Write a direct-variation relationship for the gallons supplied by each pump. Then find the amount of time that it takes for each pump to fill the tank to capacity if each works alone.
a. pump 1 **$y = 5t$, 40 minutes**
b. pump 2 **$y = 10t$, 20 minutes**
c. pump 3 **$y = 15t$, $13\frac{1}{3}$ minutes**



2. On certain occasions, some pumps are turned on at the same time. Write a direct-variation relationship for each combination of pumps operating simultaneously.
a. pump 1 and pump 2 **$y = 15t$**
b. pump 2 and pump 3 **$y = 25t$**
c. pump 1, pump 2, and pump 3 **$y = 30t$**

3. At noon, the tank contains 500 gallons of solution. At noon, all pumps, and the discharge pump, are turned on simultaneously.
a. Write an equation for the quantity, Q, of solution t minutes after noon. **$Q = 500 + 30t - 35t$**
b. Is the tank filling or emptying? At what rate is this happening? **emptying at 5 gallons per minute**
c. After how many minutes will the tank be full or empty? **after 100 minutes, the tank will be empty**

4. At noon, the tank contains 500 gallons of solution and all feeder pumps are turned on. At 12:06 p.m., the discharge pump is turned on.
a. Write an equation or equations for the quantity, Q, of solution t minutes after noon. **$Q = 500 + 30(t + 6) - 35t$, or $Q = 680 - 5t$**
b. How many minutes after noon will the tank either be full or be empty? **after 136 minutes, the tank will be empty**

TEST PREP DOCTOR Students might eliminate the correct choice **F** in **Exercise 42** because they do not understand that the constant of variation can be 1.

Journal

Have students explain how a rate of change, a slope, and a constant of variation are related.

ALTERNATIVE ASSESSMENT

Have students write two equations of direct variation involving common conversion factors, such as 12 inches:1 foot. Then have them graph the equations and identify the constants of variation.

Power Presentations with PowerPoint®

4-5 Lesson Quiz

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

1. $2y = 6x$ yes; 3

2. $3x = 4y - 7$ no

Tell whether each relationship is a direct variation. Explain.

3.

x	−4	0	8
y	2	3	5

no; $\frac{2}{-4} \neq \frac{3}{0} \neq \frac{5}{8}$

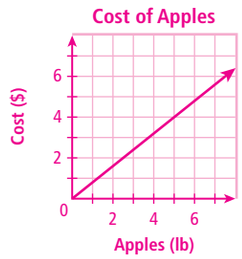
4.

x	1	3	7
y	4.3	12.9	30.1

yes; $\frac{4.3}{1} = \frac{12.9}{3} = \frac{30.1}{7}$

5. The value of y varies directly with x, and $y = -8$ when $x = 20$. Find y when $x = -4$. 1.6

6. Apples cost \$0.80 per pound. Write a direct variation equation to describe the cost y of x pounds of apples. Then graph. $y = 0.8x$



Also available online

SECTION 4A
MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills in a real-world format.

INTERACTIVE **Online Edition**

Answers

1. For table, see Additional Answers in the back of this book.

2–5, 7. See Additional Answers in the back of this book.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Characteristics of Linear Functions

Heart Health People who exercise need to be aware of their maximum heart rate.

1. One way to estimate your maximum heart rate m is to subtract 85% of your age in years from 217. Create a table of values that shows the maximum heart rates for people ages 13 to 18. Then write an equation to describe the data in the table. $m = 217 - 0.85a$
2. Use your table from Problem 1 to graph the relationship between age and maximum heart rate. What are the intercepts? What is the slope?
3. What do the intercepts represent in this situation?
4. What does the slope represent? Explain why the slope is negative.
5. Another formula for estimating maximum heart rate is $m = 206.3 - 0.711a$, where a represents age in years. Describe how this equation is different from your equation in Problem 1. Include slope and intercepts in your description.
6. Which equation gives a higher maximum heart rate for people ages 75 and younger? the first equation
7. To be exercising in your *aerobic training zone* means that your heart rate is 70% to 80% of your maximum heart rate. Write two equations that someone could use to estimate the range of heart rates that are within his or her aerobic training zone. Use your equation for maximum heart rate from Problem 1.

(t)Victoria Smith/HMH; (br)© Rick Gomez/CORBIS; (t)© age fotostock / SuperStock

INTERVENTION

Scaffolding Questions

1. What are the independent and dependent variables? independent: age; dependent: maximum heart rate
2. How can you find the slope? Possible answer: Once you know the intercepts, use (255.29, 0) and (0, 217) in the slope formula.
3. What does $a = 0$ represent? age of 0 yr What does $m = 0$ represent? max. heart rate of 0 beats/min Are these values reasonable in this situation? no
4. Does the maximum heart rate increase or decrease with age? How do you know? Decrease; the graph falls from left to right.

5–7. Discuss the variables, intercepts, and slopes in the formulas described.

Extension

Find your maximum heart rate using the method given in **Problem 5.** Then find the range of heart rates that are within your aerobic training zone using this method and the information given in **Problem 7.** Possible answer: For age 14, the maximum rate is about 196 beats/min, and the range is about 137–157 beats/min.

READY TO GO ON?

CHAPTER 4

SECTION 4A

Quiz for Lessons 4-1 Through 4-5

4-1 Identifying Linear Functions

Tell whether the given ordered pairs satisfy a linear function. Explain.

1. 1. No; a constant change of +1 in *x* corresponds to diff. changes in *y*.

x	−2	−1	0	1	2
y	1	0	1	4	9

2. $\{(-3, 8), (-2, 6), (-1, 4), (0, 2), (1, 0)\}$

Yes; a constant change of +1 in *x* corresponds to a constant change of −2 in *y*.

4-2 Using Intercepts

Use intercepts to graph the line described by each equation.

3. $2x - 4y = 16$

4. $-3y + 6x = -18$

5. $y = -3x + 3$

4-3 Rate of Change and Slope

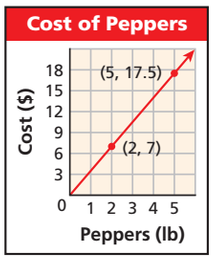
6. The chart gives the amount of water in a rain gauge in inches at various times. Graph the data and show the rates of change.

Time (h)	1	2	3	4	5
Rain (in.)	0.2	0.4	0.7	0.8	1.0

4-4 The Slope Formula

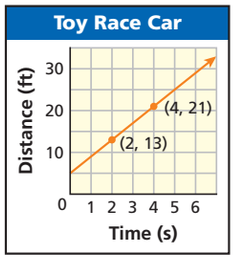
Find the slope of each line. Then tell what the slope represents.

7.



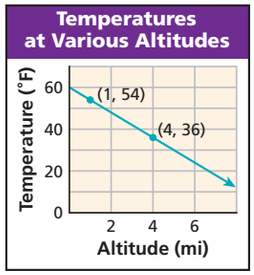
3.5; peppers cost $3.50/lb.

8.



4; speed is 4 ft/s.

9.



−6; temp. decreases 6 °F/mi.

4-5 Direct Variation

Tell whether each relationship is a direct variation. If so, identify the constant of variation.

10.

x	1	4	8	12
y	3	6	10	14

no

11.

x	−6	−2	0	3
y	−3	−1	0	1.5

yes; $\frac{1}{2}$

READY TO GO ON?

SECTION 4A

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

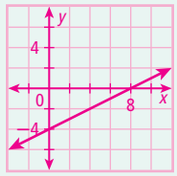
Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

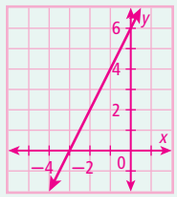
***Ready to Go On?* Online**

Answers

3.



4.



5, 6. See Additional Answers.

READY TO GO ON?

Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 4A: Worksheets	*Ready to Go On? Intervention*, Section 4A: Online	
Lesson 4-1	4-1 Intervention	Activity 4-1	Diagnose and Prescribe Online
Lesson 4-2	4-2 Intervention	Activity 4-2	
Lesson 4-3	4-3 Intervention	Activity 4-3	
Lesson 4-4	4-4 Intervention	Activity 4-4	
Lesson 4-5	4-5 Intervention	Activity 4-5	

***Ready to Go On? Enrichment*, Section 4A**
Worksheets
Online

SECTION 4B

Using Linear Functions

One-Minute Section Planner

Lesson	Lab Resources	Materials
Lesson 4-6 Slope-Intercept Form • Write a linear equation in slope-intercept form. • Graph a line using slope-intercept form. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab (A) Algebra Lab (B)	**Optional** graphing calculator, masking tape, rope or string, graph paper
Lesson 4-7 Point-Slope Form • Graph a line and write a linear equation using point-slope form. • Write a linear equation given two points. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		
Technology Lab Graph Linear Functions • Use a graphing calculator to graph lines. ☐ SAT-10 ☐ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 4-8 Line of Best Fit • Determine a line of best fit for a set of linear data. • Determine and interpret the correlation coefficient. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests		**Required** graphing calculator graph paper
Lesson 4-9 Slopes of Parallel and Perpendicular Lines • Identify and graph parallel and perpendicular lines. • Write equations to describe lines parallel or perpendicular to a given line. ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT ☑ SAT Subject Tests	***Algebra Lab Activities*** Algebra Lab	**Required** graphing calculator
Technology Lab The Family of Linear Functions • Use a graphing calculator to explore families of functions. ☐ SAT-10 ☐ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests	***Technology Lab Activities*** Lab Recording Sheet	**Required** graphing calculator
Lesson 4-10 Transforming Linear Functions • Describe how changing slope and *y*-intercept affect the graph of a linear function. ☐ SAT-10 ☑ NAEP ☐ ACT ☑ SAT ☑ SAT Subject Tests		**Optional** graphing calculator, index cards, graph paper
Extension Absolute-Value Functions • Graph absolute-value functions • Identify characteristics of absolute-value functions and their graphs. ☑ SAT-10 ☐ NAEP ☑ ACT ☐ SAT ☐ SAT Subject Tests		**Optional** tracing paper

Note: If NAEP is checked, the content is tested on either the Grade 8 or the Grade 12 NAEP assessment.

MK = *Manipulatives Kit*

Selected Answers in Student Edition

The answers to the following exercises are included in the Selected Answers section at the back of the Student Edition textbook. All answers for the Check it Out! exercises in each lesson and for the Chapter Study Guide: Review are included in the Student Edition.

Lesson 6: 1, 5, 9, 11, 13a, 13b, 13c, 15, 17, 19, 23, 29, 31, 33, 37, 39, 41

Lesson 7: 1, 3, 7, 9, 11, 13, 15, 17, 19, 23, 25, 27, 31, 33, 35, 41, 43a, 43b, 43c, 47, 49, 53a, 53b, 53c, 55, 57

Lesson 8: 1, 3, 5, 7, 9, 13a, 13b, 13c, 13d, 13e, 15a, 15b, 15c, 15d, 17, 19a, 21, 23, 25, 27, 29, 31, 33

Lesson 9: 1, 3, 5, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 51a, 51b, 53, 57

Lesson 10: 1, 3, 7, 9, 13, 17, 23, 27, 31, 37, 49, 41, 43, 45a, 45b, 45c, 49

Math Background

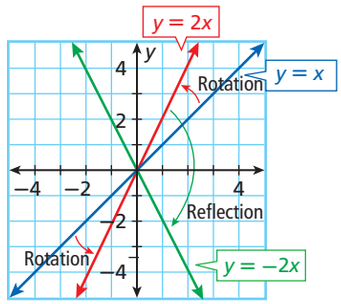
COMMON CORE PROFESSIONAL DEVELOPMENT

MORE ON LINEAR FUNCTIONS

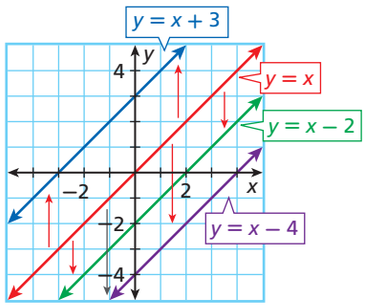
Lessons 4-6, 4-7, 4-10

Any linear function may be written in slope-intercept form, $y = mx + b$, where m is the slope of the line and b is the y-intercept. It is useful to understand how m and b affect the appearance of the line.

For nonnegative values of m, the graph of $y = mx$ is a *rotation* of the graph of $y = x$. The greater the value of m, the steeper the resulting line. The figure shows how $y = 2x$ represents a rotation of $y = x$. When m is negative, the line is reflected across the x-axis, as seen in the graph of $y = -2x$.



In general, the graph of $y = mx + b$ is a *vertical translation* of the graph of $y = mx$. If b is positive, the translation is upward by b units. If b is negative, the translation is downward by $|b|$ units. Comparing the graphs of $y = x$, $y = x + 3$, $y = x - 2$, and $y = x - 4$ illustrates this.



Putting all of these ideas together shows that the graph of every linear function is the result of one or more transformations of the graph of the *parent function* $y = x$. For example, to get the graph of $y = -5x + 3$, the graph of $y = x$ is first rotated, then reflected across the x-axis, and then translated upward by 3 units.

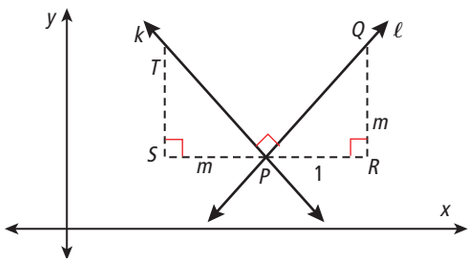
PERPENDICULAR LINES

Lesson 4-9

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 (or, equivalently, if and only if their slopes are opposite reciprocals). Students should understand the logic of the phrase *if and only if.* It means that the statement is true "in both directions": if nonvertical lines are perpendicular, then the product of their slopes is -1; if the product of the slopes is -1, then the lines are perpendicular.

Students should also be aware that they will have to accept this result as true until they are able to prove it in a geometry course. The main steps in the proof that nonvertical perpendicular lines have slopes whose product is -1 are outlined below.

Assume perpendicular lines k and ℓ intersect at P and that line ℓ has positive slope m. Then it is possible to construct a right triangle PQR that has its hypotenuse along line ℓ and has legs of length m and 1 as shown. Extend $\overline{PR}$ through P and mark off a length of m along the line (at point S). Construct a perpendicular at S to form right triangle TPS. Basic facts from geometry and the *ASA* triangle congruence theorem show that $\triangle PQR \cong \triangle TPS$. Thus, $\overline{TS} \cong \overline{PR}$ and so $TS = 1$. The slope of $\overline{TP}$ must be $-\frac{1}{m}$, and $-\frac{1}{m} \cdot m = -1$.



4-6 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Write a linear equation in slope-intercept form.

Graph a line using slope-intercept form.

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Find each *y*-intercept.

1. $y = 3x + 2$ 2

2. $5x - 3y = 12$ -4

Find each slope.

3. $y = \frac{2}{3}x$ $\frac{2}{3}$

4. $6x + 2y = 6$ -3

Solve each equation for *y*.

5. $4x + 2y = 10$ $y = -2x + 5$

6. $3x + 2 = 6y$ $y = \frac{1}{2}x + \frac{1}{3}$

Also available online

Math Humor

Q: Why was the scientist smiling as he worked?

A: His graph had a great plot!

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

CC.9-12.A.CED.3 Represent constraints by equations or inequalities, … and interpret solutions as viable or non-viable options in a modeling context.*

Also **CC.9-12.F.IF.7*, CC.9-12.F.IF.6*, CC.9-12.F.BF.1*, CC.9-12.F.LE.2***

Learn It Online
State Resources Online

COMMON CORE

4-6

Slope-Intercept Form

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities…* *Also* **CC.9-12.A.CED.3*, CC.9-12.F.IF.7*, CC.9-12.F.IF.6*, CC.9-12.F.BF.1*, CC.9-12.F.LE.2***

Objectives
Write a linear equation in slope-intercept form.

Graph a line using slope-intercept form.

Who uses this?
Consumers can use slope-intercept form to model and calculate costs, such as the cost of renting a moving van. (See Example 4.)

You have seen that you can graph a line if you know two points on the line. Another way is to use the slope of the line and the point that contains the *y*-intercept.

EXAMPLE 1 Graphing by Using Slope and *y*-intercept

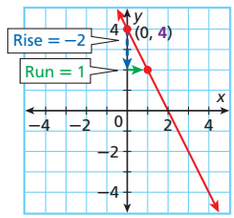
Graph the line with slope −2 and *y*-intercept 4.

Step 1 The *y*-intercept is 4, so the line contains (0, 4). Plot (0, 4).

Step 2 Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{1}$

Count **2 units down** and **1 unit right** from (0, 4) and plot another point.

Step 3 Draw the line through the two points.

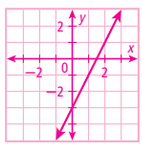


Writing Math
Any integer can be written as a fraction with 1 in the denominator.
$-2 = \frac{-2}{1}$

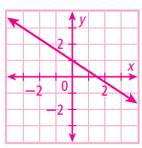
CHECK IT OUT! **Graph each line given the slope and *y*-intercept.**

1a. slope = 2, *y*-intercept = −3 **1b.** slope $= -\frac{2}{3}$, *y*-intercept = 1

1a.



1b.



If you know the slope of a line and the *y*-intercept, you can write an equation that describes the line.

Step 1 If a line has slope 2 and the *y*-intercept is 3, then $m = 2$ and $(0, 3)$ is on the line. Substitute these values into the slope formula.

Slope formula → $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad 2 = \frac{y - 3}{x - 0}$ ←*Since you don't know (x_2, y_2), use (x, y).*

Step 2 Solve for *y*: $2 = \frac{y - 3}{x - 0}$

$2 = \frac{y - 3}{x}$ *Simplify the denominator.*

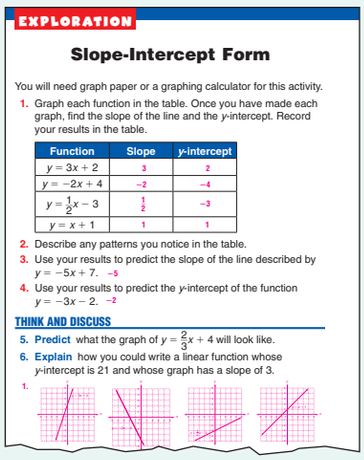
$2 \cdot x = \left(\frac{y - 3}{x}\right) \cdot x$ *Multiply both sides by x.*

$2x = y - 3$

$+3 \quad +3$ *Add 3 to both sides.*

$2x + 3 = y$, or $y = 2x + 3$

1 Introduce



EXPLORATION

Slope-Intercept Form

You will need graph paper or a graphing calculator for this activity.

1. Graph each function in the table. Once you have made each graph, find the slope of the line and the *y*-intercept. Record your results in the table.

Function	Slope	*y*-intercept
$y = 3x + 2$	3	2
$y = -2x + 4$	−2	4
$y = \frac{1}{2}x - 3$	$\frac{1}{2}$	−3
$y = x + 1$	1	1

2. Describe any patterns you notice in the table.
3. Use your results to predict the slope of the line described by $y = -5x + 7$. −5
4. Use your results to predict the *y*-intercept of the function $y = -3x - 2$. −2

THINK AND DISCUSS

5. **Predict** what the graph of $y = \frac{2}{3}x + 4$ will look like.
6. **Explain** how you could write a linear function whose *y*-intercept is 21 and whose graph has a slope of 3.

1.

Motivate

Ask students how they would graph the function $y = 2x + 3$. Possible answers: generate ordered pairs; use intercepts

Then tell students that in this lesson, they will learn a quick way to graph linear functions. It does not involve generating ordered pairs and uses only one intercept.

Explorations and answers are provided in your online resources.

Slope-Intercept Form of a Linear Equation

If a line has slope m and the y-intercept is b, then the line is described by the equation $y = mx + b$.

Any linear equation can be written in slope-intercept form by solving for y and simplifying. In this form, you can immediately see the slope and y-intercept. Also, you can quickly graph a line when the equation is written in slope-intercept form.

EXAMPLE 2 Writing Linear Equations in Slope-Intercept Form

Write the equation that describes each line in slope-intercept form.

A slope $= \frac{1}{3}$, y-intercept $= 6$

$y = mx + b$ — *Substitute the given values for m and b.*

$y = \frac{1}{3}x + 6$ — *Simplify if necessary.*

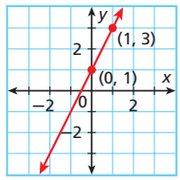
B slope $= 0$, y-intercept $= -5$

$y = mx + b$

$y = 0x + (-5)$

$y = -5$

C



Step 1 Find the y-intercept. The graph crosses the y-axis at $(0, 1)$, so $b = 1$.

Step 2 Find the slope. The line contains the points $(0, 1)$ and $(1, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — *Use the slope formula.*

$m = \frac{3 - 1}{1 - 0} = \frac{2}{1} = 2$ — *Substitute (0, 1) for (x_1, y_1) and (1, 3) for (x_2, y_2).*

Step 3 Write the equation.

$y = mx + b$ — *Write the slope-intercept form.*

$y = 2x + 1$ — *Substitute 2 for m and 1 for b.*

D slope $= 4$, $(2, 5)$ is on the line

Step 1 Find the y-intercept.

$y = mx + b$ — *Write the slope-intercept form.*

$5 = 4(2) + b$ — *Substitute 4 for m, 2 for x, and 5 for y.*

$5 = 8 + b$ — *Solve for b. Since 8 is added to b, subract 8 from both sides to undo the addition.*

$-8 \quad -8$

$-3 = b$

Step 2 Write the equation.

$y = mx + b$ — *Write the slope-intercept form.*

$y = 4x + (-3)$ — *Substitute 4 for m and −3 for b.*

$y = 4x - 3$

CHECK IT OUT! **Write the equation that describes each line in slope-intercept form.**

2a. slope $= -12$, y-intercept $= -\frac{1}{2}$

2b. slope $= 1$, y-intercept $= 0$

2c. slope $= 8$, $(-3, 1)$ is on the line.

2a. $y = -12x - \frac{1}{2}$

2b. $y = x$

2c. $y = 8x + 25$

2 Teach

Guided Instruction

Remind students that slope is the ratio of rise over run. Graph a line such as $y = -\frac{1}{2}x + 2$ in two ways, once using a slope of $\frac{-1}{2}$ and once using a slope of $\frac{1}{-2}$, to show that they result in the same line. In **Example 3,** remind students that they can quickly check that their graph is reasonable by looking at the slope. Lines with positive slopes rise from left to right, and lines with negative slopes fall from left to right.

Reaching All Learners

Through Kinesthetic Experience

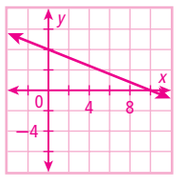
Use masking tape to outline a coordinate plane on a floor of square tiles. Then give a pair of students a length of rope. Announce an equation in slope-intercept form, and have the two students move around on the plane so that when they hold the rope taut, it represents the line described by the given equation, with each of them as two points on the line. A third student can check that these two points satisfy the equation.

Power Presentations with PowerPoint®

Additional Examples

Example 1

Graph the line with slope $-\frac{2}{5}$ and y-intercept 4.



Example 2

Write the equation that describes each line in slope-intercept form.

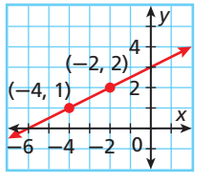
A. slope $= \frac{1}{4}$; y-intercept $= 4$

$y = \frac{1}{4}x + 4$

B. slope $= -9$; y-intercept $= -\frac{5}{4}$

$y = -9x - \frac{5}{4}$

C.



$y = 3x + \frac{1}{2}$

D. slope $= 2$; $(3, 4)$ is on the line. $y = 2x - 2$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 1

- What is the first step when graphing a line given the slope and y-intercept?

EXAMPLE 2

- What does b represent?
- What do the equation and graph look like if $b = 0$?

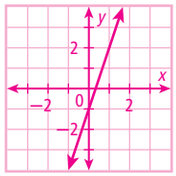
Power Presentations with PowerPoint®

Additional Examples

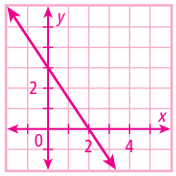
Example 3

Write each equation in slope-intercept form. Then graph the line described by the equation.

A. $y = 3x - 1$ $y = 3x - 1$



B. $2y + 3x = 6$ $y = -\frac{3}{2}x + 3$



Also available online

Questioning Strategies

EXAMPLE 3

- Does the coefficient of x in a linear equation always equal the slope? Explain.

Multiple Representations Remind students that they could use the intercepts to graph the equation in **Example 3B.**

EXAMPLE 3 Using Slope-Intercept Form to Graph

Write each equation in slope-intercept form. Then graph the line described by the equation.

A $y = 4x - 3$

$y = 4x - 3$ is in the form $y = mx + b$.

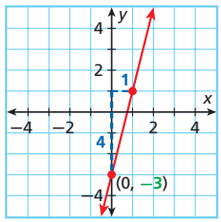
slope: $m = 4 = \frac{4}{1}$

y-intercept: $b = -3$

Step 1 Plot $(0, -3)$.

Step 2 Count **4 units up** and **1 unit right** and plot another point.

Step 3 Draw the line connecting the two points.



B $y = -\frac{2}{3}x + 2$

$y = -\frac{2}{3}x + 2$ is in the form $y = mx + b$.

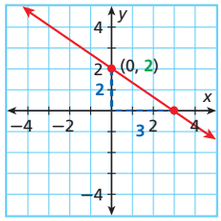
slope: $m = -\frac{2}{3} = \frac{-2}{3}$

y-intercept: $b = 2$

Step 1 Plot $(0, 2)$

Step 2 Count **2 units down** and **3 units right** and plot another point.

Step 3 Draw the line connecting the two points.



C $3x + 2y = 8$

Step 1 Write the equation in slope-intercept form by solving for y.

$3x + 2y = 8$

$-3x \quad -3x$ *Subtract 3x from both sides.*

$2y = 8 - 3x$

$\frac{2y}{2} = \frac{8 - 3x}{2}$ *Since y is multiplied by 2, divide both sides by 2.*

$y = 4 - \frac{3}{2}x$ $\frac{3x}{2} = \frac{3}{2}x$

$y = -\frac{3}{2}x + 4$ *Write the equation in the form y = mx + b.*

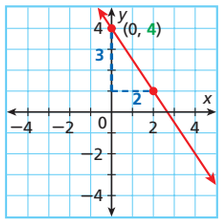
Step 2 Graph the line.

$y = -\frac{3}{2}x + 4$ is in the form $y = mx + b$.

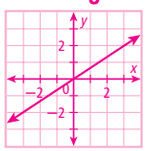
slope: $m = -\frac{3}{2} = \frac{-3}{2}$

y-intercept: $b = 4$

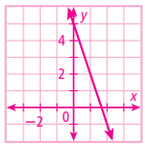
- Plot $(0, 4)$.
- Then count **3 units down** and **2 units right** and plot another point.
- Draw the line connecting the two points.



3a. $y = \frac{2}{3}x$



3b. $y = -3x + 5$

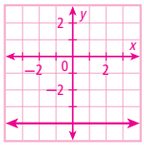


Helpful Hint

To divide $(8 - 3x)$ by 2, you can multiply by $\frac{1}{2}$ and use the Distributive Property.

$$\frac{8 - 3x}{2} = \frac{1}{2}(8 - 3x)$$
$$= \frac{1}{2}(8) + \frac{1}{2}(-3x)$$
$$= 4 - \frac{3}{2}x$$

3c. $y = -4$



CHECK IT OUT! **Write each equation in slope-intercept form. Then graph the line described by the equation.**

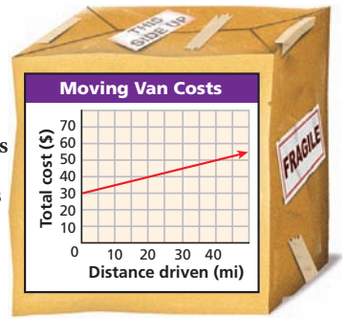
3a. $y = \frac{2}{3}x$ **3b.** $6x + 2y = 10$ **3c.** $y = -4$

Inclusion Students may confuse rise and run when counting slope on a graph. Draw $\updownarrow$ on the board to help them remember which number goes up or down and which goes left or right.

Diversity In **Check it Out 4,** the term *caterer* may be unfamiliar to some students. Explain that a caterer prepares food that is served at a special event, such as a wedding.

ENGLISH LANGUAGE LEARNERS

EXAMPLE 4 *Consumer Application*



To rent a van, a moving company charges $30.00 plus $0.50 per mile. The cost as a function of the number of miles driven is shown in the graph.

a. Write an equation that represents the cost as a function of the number of miles.

Cost	is	$0.50 per mile	times	miles	plus	$30.00
y	=	0.5	•	x	+	30

An equation is $y = 0.5x + 30$.

b. Identify the slope and *y*-intercept and describe their meanings.

The *y*-intercept is 30. This is the cost for 0 miles, or the initial fee of $30.00.

The slope is 0.5. This is the rate of change of the cost: $0.50 per mile.

c. Find the cost of the van for 150 miles.

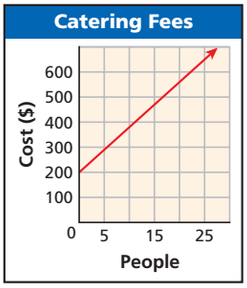
$y = 0.5x + 30$

$= 0.5(150) + 30 = 105$ *Substitute 150 for x in the equation.*

The cost of the van for 150 miles is $105.

CHECK IT OUT!

4. A caterer charges a $200 fee plus $18 per person served. The cost as a function of the number of guests is shown in the graph.



a. Write an equation that represents the cost as a function of the number of guests. $y = 18x + 200$

b. Identify the slope and *y*-intercept and describe their meanings.

4b. slope: 18; cost per person; *y*-int: 200; fee

c. Find the cost of catering an event for 200 guests. **$3800**

MATHEMATICAL PRACTICES

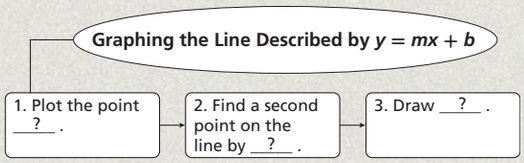
THINK AND DISCUSS

1. If a linear function has a *y*-intercept of *b*, at what point does its graph cross the *y*-axis?

2. Where does the line described by $y = 4.395x - 23.75$ cross the *y*-axis?

3. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note



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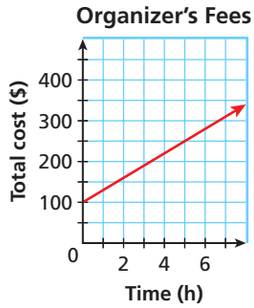
Additional Examples

Example 4

A closet organizer charges a $100 initial consultation fee plus $30 per hour. The cost as a function of the number of hours worked is graphed below.



a. Write an equation that represents the cost as a function of the number of hours. $y = 30x + 100$

b. Identify the slope and *y*-intercept and describe their meanings. *y*-int.: 100; the cost for 0 h, or the initial fee of $100. Slope: 30; the rate of change of the cost: $30/h.

c. Find the cost if the organizer works 12 hours. $460

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- Why doesn't the graph start at the origin?

3 Close

Summarize

Review slope-intercept form, reminding students that an integer slope can be written with a denominator of 1. Review how to write an equation of a line in slope-intercept form when given the slope and *y*-intercept or when given the slope and a point on the line.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. $(0, b)$
2. $(0, -23.75)$
3. See Additional Answers.

4-6 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

Assignment Guide

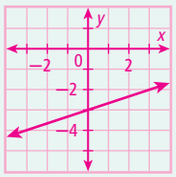
Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 14–22
Average 14–22, 42
Advanced 14–22, 42

If you finished Examples **1–4**
Basic 14–27, 32–41
Average 14–31, 35–42
Advanced 14–31, 35–44

Homework Quick Check
Quickly check key concepts.
Exercises: 14, 18, 20, 24, 26, 36

Answers

1. 

2–4. See Additional Answers.

10–12. For graphs, see Additional Answers.

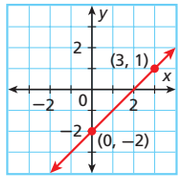
14–17. See Additional Answers.

GUIDED PRACTICE

SEE EXAMPLE 1 Graph each line given the slope and *y*-intercept.

1. slope $= \frac{1}{3}$, *y*-intercept $= -3$
2. slope $= 0.5$, *y*-intercept $= 3.5$
3. slope $= 5$, *y*-intercept $= -1$
4. slope $= -2$, *y*-intercept $= 2$

SEE EXAMPLE 2 Write the equation that describes each line in slope-intercept form.

5. 

$y = x - 2$

6. slope $= 8$, *y*-intercept $= 2$ $y = 8x + 2$
7. slope $= 0$, *y*-intercept $= -3$ $y = -3$
8. slope $= 5$, $(2, 7)$ is on the line. $y = 5x - 3$
9. slope $= -2$, $(1, -3)$ is on the line. $y = -2x - 1$

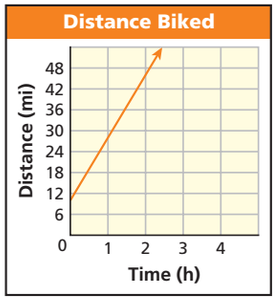
SEE EXAMPLE 3 Write each equation in slope-intercept form. Then graph the line described by the equation.

10. $y = \frac{2}{5}x - 6$ $y = \frac{2}{5}x - 6$
11. $3x - y = 1$ $y = 3x - 1$
12. $2x + y = 4$ $y = -2x + 4$

SEE EXAMPLE 4

13. Helen is in a bicycle race. She has already biked 10 miles and is now biking at a rate of 18 miles per hour. Her distance as a function of time is shown in the graph.
    a. Write an equation that represents the distance Helen has biked as a function of time. $y = 18x + 10$
    b. Identify the slope and *y*-intercept and describe their meanings.
    c. How far will Helen have biked after 2 hours? 46 mi

13b. slope: 18; Helen's speed; *y*-int.: 10; dist. she has already biked



PRACTICE AND PROBLEM SOLVING

Independent Practice

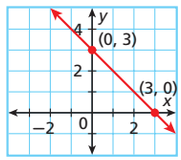
For Exercises	See Example
14–17	1
19–22	2
23–25	3
26	4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Graph each line given the slope and *y*-intercept.

14. slope $= \frac{1}{4}$, *y*-intercept $= 7$
15. slope $= -6$, *y*-intercept $= -3$
16. slope $= 1$, *y*-intercept $= -4$
17. slope $= -\frac{4}{5}$, *y*-intercept $= 6$

Write the equation that describes each line in slope-intercept form.

18. 

$y = -x + 3$

19. slope $= 5$, *y*-intercept $= -9$ $y = 5x - 9$
20. slope $= -\frac{2}{3}$, *y*-intercept $= 2$ $y = -\frac{2}{3}x + 2$
21. slope $= -\frac{1}{2}$, $(6, 4)$ is on the line. $y = -\frac{1}{2}x + 7$
22. slope $= 0$, $(6, -8)$ is on the line. $y = -8$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 13, 26–27, 36

Reason abstractly and quantitatively. Exercises 43

Construct viable arguments and critique the reasoning of others. Exercises 28–31, 35, 44

Look for and make use of structure. Exercises 32–34, 42

Learn It Online
State Resources Online

READING STRATEGIES

The procedure outlined below shows how to graph a line using slope-intercept form.

Graph the line described by $2x + y = 4$.

Step 1: Rewrite in slope-intercept form. $2x + y = 4$; $-2x \quad -2x$; $y = -2x + 4$

Step 2: Identify the slope and *y*-intercept. $y = mx + b$; $m = -2 = \frac{-2}{1}$; $b = 4$

Step 3: Plot the point $(0, b)$.

Step 4: Plot a second point using the slope. Down 2, Right 1

Step 5: Draw a line through the points.

Answer the following.

1. What is the benefit of always writing slope as a fraction?
   With a fraction, you have a "rise" and "run" for graphing.
2. What point would you plot first if $b = -8$? $(0, -8)$

Identify the slope and *y*-intercept for each equation.

3. $y = 5x + 12$ m: 5 b: 12
4. $y = -3x$ m: −3 b: 0
5. $y = x - 4$ m: 1 b: −4
6. $3y = x + 9$ m: $\frac{1}{3}$ b: 12

Graph the line described by each equation.

7. $3x + y = 2$
8. $x - 2y = 6$

RETEACH

An equation is in **slope-intercept form** if it is written as: $y = mx + b$. *m* is the slope. *b* is the *y*-intercept.

A line has a slope of −4 and a *y*-intercept of 3. Write the equation in slope-intercept form.

$y = mx + b$ Substitute the given values for *m* and *b*.
$y = -4x + 3$

A line has a slope of 2. The ordered pair (3, 1) is on the line. Write the equation in slope-intercept form.

Step 1: Find the *y*-intercept.
$y = mx + b$
$y = 2x + b$ Substitute the given value for *m*.
$1 = 2(3) + b$ Substitute the given values for *x* and *y*.
$1 = 6 + b$ Solve for *b*.
$-6 \quad -6$
$-5 = b$

Step 2: Write the equation.
$y = mx + b$
$y = 2x - 5$ Substitute the given value for *m* and the value you found for *b*.

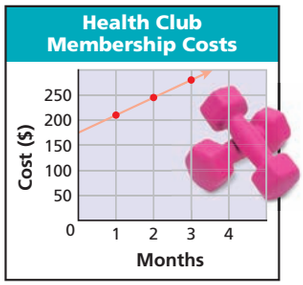
Write the equation that describes each line in slope-intercept form.

1. slope $= \frac{1}{4}$, *y*-intercept $= 3$ $y = \frac{1}{4}x + 3$
2. slope $= -5$, *y*-intercept $= 0$ $y = -5x$
3. slope $= 7$, *y*-intercept $= -2$ $y = 7x - 2$
4. slope is 3, (4, 6) is on the line. $y = 3x - 6$
5. slope is $\frac{1}{2}$, $(-2, 8)$ is on the line. $y = \frac{1}{2}x + 9$
6. slope is -1, $(5, -2)$ is on the line. $y = -x + 3$

Write each equation in slope-intercept form. Then graph the line described by the equation.

23. $-\frac{1}{2}x + y = 4$ **24.** $\frac{2}{3}x + y = 2$ **25.** $2x + y = 8$

26. Fitness Pauline's health club has an enrollment fee of $175 and costs $35 per month. Total cost as a function of number of membership months is shown in the graph.



a. Write an equation that represents the total cost as a function of months. $y = 35x + 175$

b. Identify the slope and *y*-intercept and describe their meanings.

c. Find the cost of one year of membership. $595

26b. slope: 35; monthly cost; *y*-int.: 175; enrollment fee

27. A company rents video games. The table shows the linear relationship between the number of games a customer can rent at one time and the monthly cost of the service.

Games Rented at One Time	1	2	3
Monthly Cost ($)	14	18	22

a. Graph the relationship.

b. Write an equation that represents the monthly cost as a function of games rented at one time. $y = 4x + 10$

Critical Thinking **Tell whether each situation is possible or impossible. If possible, draw a sketch of the graphs. If impossible, explain.**

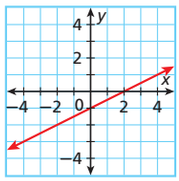
28. Two different lines have the same slope. possible

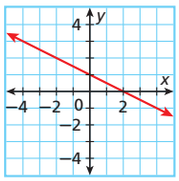
29. Two different linear functions have the same *y*-intercept. possible

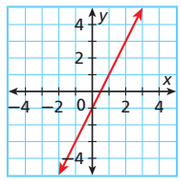
30. Two intersecting lines have the same slope. impossible

31. A linear function does not have a *y*-intercept. impossible

Match each equation with its corresponding graph.

32.  B

33.  C

34.  A

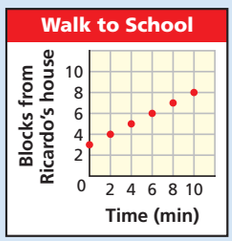
35. Possible answer: $x = -4$; no; because it has an undef. slope and no *y*-int.

A. $y = 2x - 1$ B. $y = \frac{1}{2}x - 1$ C. $y = -\frac{1}{2}x + 1$

35. Write About It Write an equation that describes a vertical line. Can you write this equation in slope-intercept form? Why or why not?

MULTI-STEP TEST PREP

36. a. Ricardo and Sam walk from Sam's house to school. Sam lives 3 blocks from Ricardo's house. The graph shows their distance from Ricardo's house as they walk to school. Create a table of these values.



b. Find an equation for the distance as a function of time.

c. What are the slope and *y*-intercept? What do they represent in this situation?

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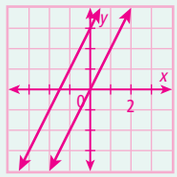
MULTI-STEP TEST PREP **Exercise 36** involves graphing and interpreting a linear equation in slope-intercept form. This exercise prepares students for the Multi-Step Test Prep.

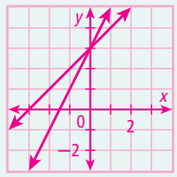
Answers

23–25, 27a. See Additional Answers.

28.



29.



30. Lines with the same slope are par. and therefore cannot intersect.

31. If a lin. func. does not have a *y*-int., then its graph does not intersect the *y*-axis. The *y*-axis is vert. so the only lines that do not intersect the *y*-axis are also vert. But vert. lines cannot be graphs of functions. All nonvert. lines will intersect the *y*-axis, so every lin. func. will have a *y*-int.

36a.

x	*y*
0	3
2	4
4	5
6	6
8	7
10	8

b. $y = \frac{1}{2}x + 3$

c. slope: $\frac{1}{2}$; boys' walking speed; *y*-int.: 3; dist. from Sam's house to Ricardo's house

PRACTICE A

PRACTICE C

PRACTICE B

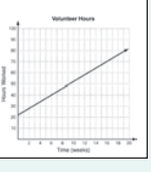
Write the equation that describes each line in slope-intercept form.

1. slope = 4; *y*-intercept = −3
   $y = 4x - 3$
2. slope = −2; *y*-intercept = 0
   $y = -2x$
3. slope = $-\frac{1}{3}$; *y*-intercept = 6
   $y = \frac{1}{3}x + 6$
4. slope = $\frac{2}{5}$; (10, 3) is on the line.
   Find the *y*-intercept $y = mx + b$
   $3 = (\frac{2}{5})10 + b$
   $3 = 4 + b$
   $-1 = b$
   Write the equation: $y = \frac{2}{5}x - 1$

Write each equation in slope-intercept form. Then graph the line described by the equation.

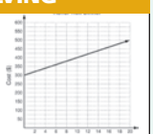
5. $y + x = 3$
   $y = -x + 3$
6. $y + 4 = \frac{4}{3}x$
   $y = \frac{4}{3}x - 4$
7. $5x - 2y = 10$
   $y = \frac{5}{2}x - 5$

8. Daniel works as a volunteer in a homeless shelter. So far, he has worked 22 hours, and he plans to continue working 3 hours per week. His hours worked as a function of time is shown in the graph.
   a. Write an equation that represents the hours Daniel will work as a function of time. $y = 3x + 22$
   b. Identify the slope and *y*-intercept and describe their meanings. slope: 3; number of hours per week; *y*-int: 22; hours already worked
   c. Find the number of hours worked after 16 weeks. 70 hours



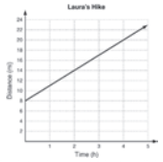
PROBLEM SOLVING

The cost of food for an honor roll dinner is $300 plus $10 per student. The cost of the food as a function of the number of students is shown in the graph. Write the correct answer.



1. Write an equation that represents the cost as a function of the number of students.
   $y = 10x + 300$
2. Identify the slope and *y*-intercept and describe their meanings.
   slope: 10, rate of the change of the cost: $10 per student; *y*-int: 300, the initial fee (the cost for 0 students)
3. Find the cost of the food for 50 students. $800

Laura is on a two-day hike in the Smoky Mountains. She hiked 8 miles on the first day and is hiking at a rate of 3 mi/h on the second day. Her total distance as a function of time is shown in the graph. Select the best answer.



4. Which equation represents Laura's total distance as a function of time?
   A $y = 3x$ C $y = 3x + 8$
   B $y = 8x$ D $y = 8x + 3$
5. What does the slope represent?
   F Laura's total distance after one day
   G Laura's total distance after two days
   H the number of miles Laura hiked per hour on the first day
   J the number of miles Laura hikes per hour on the second day
6. What does the *y*-intercept represent?
   A Laura's total distance after one day
   B Laura's total distance after two days
   C the number of miles Laura hiked per hour on the first day
   D the number of miles Laura hikes per hour on the second day
7. What will be Laura's total distance if she hikes for 6 hours on the second day?
   F 14 miles H 26 miles
   G 18 miles J 28 miles

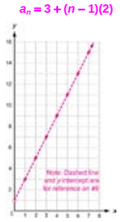
CHALLENGE

In Lesson 4-6, you learned about arithmetic sequences. In this activity, you will see that arithmetic sequences and linear equations are closely related.

For 1–7, consider this arithmetic sequence: 3, 5, 7, 9, . . .

1. What is the first term a_1 of the sequence? $a_1 = 3$
2. What is the common difference *d* of the sequence? $d = 2$
3. Use what you learned in Lesson 4-6 to write a formula for the *n*th term of the sequence. $a_n = 3 + (n - 1)(2)$
4. Complete this table, where *x* is the term number and *y* is the term.

x	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15



5. Graph the ordered pairs from problem 4. Does it make sense to connect the points with a line? Explain.
   No, because the domain of the sequence is restricted to natural numbers: {1, 2, 3, 4, ...}.
6. Write the equation that describes the line that would pass through the points in your graph. Use slope-intercept form.
   $y = 2x + 1$
7. Compare the formula in problem 3 with the equation in problem 6.
   a. What part of the equation relates to the common difference in the formula?
      The slope is the same as the common difference ($m = d = 2$).
   b. What is the relationship between the first term in the formula and the *y*-intercept in the equation?
      The *y*-intercept is the same as the first term less the common difference ($b = a_1 - d = 1$).
8. The *n*th term of an arithmetic sequence is given by the formula $a_n = 5 + (n - 1)(-3)$. If you were to graph ordered pairs where *x* is the term number and *y* is the term, what linear equation would describe the line that passes through the points?
   $y = -3x + 8$; $m = d = -3$ and $b = a_1 - d = 5 - (-3) = 8$
9. An arithmetic sequence is graphed on a coordinate plane. The equation of the line that passes through the points is $y = 5x - 1$. What is the formula for the *n*th term?
   $a_n = 4 + (n - 1)(5)$; $d = m = 5$ and $a_1 = b + d = -1 + 5 = 4$

TEST PREP DOCTOR In **Exercise 37,** students who chose **C** chose a function whose graph has the same slope, but not the same *y*-intercept. Students who chose **D** may have forgotten to divide by −2.

Journal

Have students describe three different ways to graph a line.

ALTERNATIVE ASSESSMENT

Have students write equations for 4 lines: one whose slope is a positive integer, one whose slope is a negative integer, and two whose slopes are fractions. Three should be in slope-intercept form, and one in standard form. Then have them trade equations with a partner, who will ensure that the above conditions are met and then graph the lines.

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4-6 Lesson Quiz

Write the equation that describes each line in slope-intercept form.

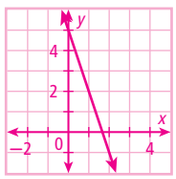
1. slope = 3, *y*-intercept = −2
$y = 3x - 2$

2. slope = 0, *y*-intercept = $\frac{1}{2}$
$y = \frac{1}{2}$

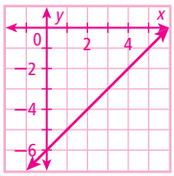
3. slope = $\frac{3}{2}$, (2, 7) is on the line.
$y = \frac{3}{2}x + 4$

Write each equation in slope-intercept form. Then graph the line described by the equation.

4. $6x + 2y = 10$ $y = -3x + 5$



5. $x - y = 6$ $y = x - 6$



Also available online

TEST PREP

37. Which function has the same *y*-intercept as $y = \frac{1}{2}x - 2$?

Ⓐ $2x + 3y = 6$ Ⓑ $x + 4y = -8$ Ⓒ $-\frac{1}{2}x + y = 4$ Ⓓ $\frac{1}{2}x - 2y = -2$

38. What is the slope-intercept form of $x - y = -8$?

Ⓕ $y = -x - 8$ Ⓖ $y = x - 8$ Ⓗ $y = -x + 8$ Ⓙ $y = x + 8$

39. Which function has a *y*-intercept of 3?

Ⓐ $2x - y = 3$ Ⓑ $2x + y = 3$ Ⓒ $2x + y = 6$ Ⓓ $y = 3x$

40. Gridded Response What is the slope of the line described by $-6x = -2y + 5$? **3**

41. Short Response Write a function whose graph has the same slope as the line described by $3x - 9y = 9$ and the same *y*-intercept as $8x - 2y = 6$. Show your work.
$y = \frac{1}{3}x - 3$

CHALLENGE AND EXTEND

42. The standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. What is the slope? What is the *y*-intercept?
$y = -\frac{A}{B}x + \frac{C}{B}$; slope $= -\frac{A}{B}$; *y*-int.: $\frac{C}{B}$

43. What value of n in the equation $nx + 5 = 3y$ would give a line with slope −2? **−6**

44. If b is the *y*-intercept of a linear function whose graph has slope m, then $y = mx + b$ describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

Statements	Reasons
1. $m = \frac{y_2 - y_1}{x_2 - x_1}$	1. Slope formula
2. $m = \frac{y - b}{x - 0}$	2. By definition, if b is the *y*-intercept, then $(0, b)$ is a point on the line. (x, y) is any other point on the line.
3. $m = \frac{y - b}{x}$	3. Any number minus 0 is the number itself.
4. m **x** $= y - b$	4. Multiplication Property of Equality (Multiply both sides of the equation by x.)
5. $mx + b = y$, or $y = mx + b$	5. Addition Property of Equality (Add b to both sides.)

COMMON CORE

4-7 Point-Slope Form

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *Also* **CC.9-12.A.CED.3***, **CC.9-12.F.IF.7***, **CC.9-12.F.BF.1***, **CC.9-12.F.LE.2***

Objectives

Graph a line and write a linear equation using point-slope form.

Write a linear equation given two points.

Why learn this?

You can use point-slope form to represent a cost function, such as the cost of placing a newspaper ad. (See Example 5.)

If you know the slope and any point on the line, you can write an equation of the line by using the slope formula. For example, suppose a line has a slope of 3 and contains $(2, 1)$. Let (x, y) be any other point on the line.

$m = \frac{y_2 - y_1}{x_2 - x_1} \longrightarrow 3 = \frac{y - 1}{x - 2}$ *Substitute into the slope formula.*

Slope formula

$3(x - 2) = \left(\frac{y - 1}{x - 2}\right)(x - 2)$ *Multiplication Property of Equality*

$3(x - 2) = y - 1$ *Simplify.*

$y - 1 = 3(x - 2)$

Know it! Note

Point-Slope Form of a Linear Equation

The line with slope m that contains the point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

EXAMPLE 1 Writing Linear Equations in Point-Slope Form

Write an equation in point-slope form for the line with the given slope that contains the given point.

A slope $= \frac{5}{2}$; $(-3, 0)$

$y - y_1 = m(x - x_1)$ *Write the point-slope form.*

$y - 0 = \frac{5}{2}[x - (-3)]$ *Substitute $\frac{5}{2}$ for m, −3 for x_1, and 0 for y_1.*

$y - 0 = \frac{5}{2}(x + 3)$ *Rewrite subtraction of negative numbers as addition.*

B slope $= -7$; $(4, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = -7(x - 4)$

C slope $= 0$; $(-2, -3)$

$y - y_1 = m(x - x_1)$

$y - (-3) = 0[x - (-2)]$

$y + 3 = 0(x + 2)$

CHECK IT OUT! **Write an equation in point-slope form for the line with the given slope that contains the given point.**

1a. slope $= 2$; $\left(\frac{1}{2}, 1\right)$ $y - 1 = 2\left(x - \frac{1}{2}\right)$

1b. slope $= 0$; $(3, -4)$ $y + 4 = 0(x - 3)$

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1 Introduce

EXPLORATION

Point-Slope Form

In this activity, you will investigate another way to write linear equations.

1a. Does the line described by $y - 4 = 5(x - 3)$ contain the point $(3, 4)$? How do you know?

b. What is the slope of the line described by $y - 4 = 5(x - 3)$?

2a. Does the line described by $y - 2 = \frac{1}{2}(x - 1)$ contain the point $(1, 2)$? How do you know?

b. What is the slope of the line described by $y - 2 = \frac{1}{2}(x - 1)$?

3a. Does the line described by $y - 6 = -3(x - 12)$ contain the point $(12, 6)$? How do you know?

b. What is the slope of the line described by $y - 6 = -3(x - 12)$?

4. Describe any patterns you notice in Problems 1–3.

THINK AND DISCUSS

5. Explain how you can use the patterns you discovered to make some predictions about the line described by $y + 2 = -2(x + 3)$.

6. Discuss what equation you think describes a line with slope 4 that contains the point $(5, 8)$. Explain your reasoning.

1a. Yes; $(3, 4)$ is a solution of $y - 4 = 5(x - 3)$. b. 5
2a. Yes; $(1, 2)$ is a solution of $y - 2 = \frac{1}{2}(x - 1)$. b. $\frac{1}{2}$
3a. Yes; $(12, 6)$ is a solution of $y - 6 = -3(x - 12)$. b. -3

Motivate

Plot the points $(-3, 2)$ and $(1, -6)$ on a coordinate grid. Ask students to find the slope of the line that contains these two points. -2 Then ask what additional information they would need to write the equation of the line using slope-intercept form. the y-intercept

Tell students that in this lesson, they will learn a new formula to write an equation of a line, using any two points on the line.

Explorations and answers are provided in your online resources.

4-7 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Graph a line and write a linear equation using point-slope form.

Write a linear equation given two points.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Find the slope of the line containing each pair of points.

1. $(0, 2)$ and $(3, 4)$ $\frac{2}{3}$

2. $(-2, 8)$ and $(4, 2)$ -1

3. $(3, 3)$ and $(12, -15)$ -2

Write the following equations in slope-intercept form.

4. $y - 5 = 3(x + 2)$ $y = 3x + 11$

5. $3x + 4y + 20 = 0$ $y = -\frac{3}{4}x - 5$

Also available online

Math Humor

Q: What did the horizontal line say to the vertical line?

A: It was nice running into you.

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

CC.9-12.A.CED.3 Represent constraints by equations or inequalities, … and interpret solutions as viable or non-viable options in a modeling context.* *Also* **CC.9-12.F.IF.7***, **CC.9-12.F.BF.1***, **CC.9-12.F.LE.2***

Learn It Online
State Resources Online

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Additional Examples

Example 1

Write an equation in point-slope form for the line with the given slope that contains the given point.

A. slope $= \frac{1}{6}$; $(5, 1)$

$y - 1 = \frac{1}{6}(x - 5)$

B. slope $= -4$; $(0, 3)$

$y - 3 = -4(x - 0)$

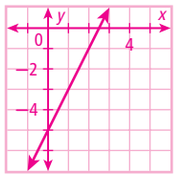
C. slope $= 1$; $(-1, -4)$

$y + 4 = 1(x + 1)$

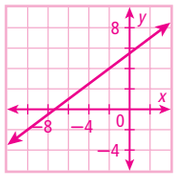
Example 2

Graph the line described by each equation.

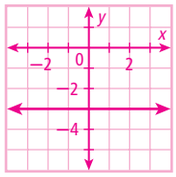
A. $y - 1 = 2(x - 3)$



B. $y - 4 = \frac{3}{4}(x + 2)$



C. $y + 3 = 0(x - 4)$



Also available online

Previously, you graphed a line given its equation in slope-intercept form. You can also graph a line when given its equation in point-slope form. Start by using the equation to identify a point on the line. Then use the slope of the line to identify a second point.

EXAMPLE 2 Using Point-Slope Form to Graph

Graph the line described by each equation.

A $y - 1 = 3(x - 1)$

$y - 1 = 3(x - 1)$ is in the form $y - y_1 = m(x - x_1)$.

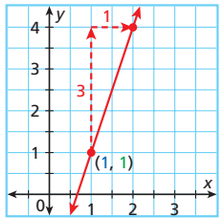
The line contains the point $(1, 1)$.

slope: $m = 3 = \frac{3}{1}$

Step 1 Plot $(1, 1)$.

Step 2 Count **3 units up** and **1 unit right** and plot another point.

Step 3 Draw the line connecting the two points.



B $y + 2 = -\frac{1}{2}(x - 3)$

Step 1 Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

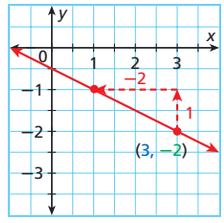
$y - (-2) = -\frac{1}{2}(x - 3)$ *Rewrite addition of 2 as subtraction of −2.*

Step 2 Graph the line.

The line contains the point $(3, -2)$.

slope: $m = -\frac{1}{2} = \frac{1}{-2}$

- Plot $(3, -2)$.
- Count 1 unit up and 2 units left and plot another point.
- Draw the line connecting the two points.

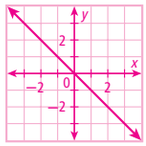


Helpful Hint

For a negative fraction, you can write the negative sign in one of three places.

$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

2a.



CHECK IT OUT! **Graph the line described by each equation.**

2a. $y + 2 = -(x - 2)$ **2b.** $y + 3 = -2(x - 1)$

EXAMPLE 3 Writing Linear Equations in Slope-Intercept Form

Write the equation that describes each line in slope-intercept form.

A **slope $= -4$, $(-1, -2)$ is on the line.**

Step 1 Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

$y - (-2) = -4[x - (-1)]$

Step 2 Write the equation in slope-intercept form by solving for y.

$y - (-2) = -4[x - (-1)]$

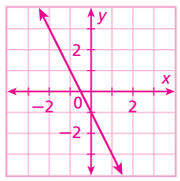
$y + 2 = -4(x + 1)$ *Rewrite subtraction of negative numbers as addition. Distribute −4 on the right side.*

$y + 2 = -4x - 4$

$-2 \quad -2$ *Subtract 2 from both sides.*

$y = -4x - 6$

2b.



Questioning Strategies

EXAMPLE 1

- How do you use the point to write the equation?

EXAMPLE 2

- How could you check that the graph is reasonable?

2 Teach

Guided Instruction

Show students that they can graph a line starting at *any* point on the line if they know the slope, by counting vertically and horizontally from that point. Show how point-slope form comes from the slope formula, and show how it also simplifies to slope-intercept form. Ask students why you might want to rewrite an equation that is in point-slope form in slope-intercept form. Possible answer: because you can quickly graph when an equation is written in slope-intercept form

Reaching All Learners

Through Auditory Cues

Have students work in pairs to quiz each other out loud about the various forms of a linear equation. Have one student state a linear equation, such as $3x + y = 8$ or $y = 2x + 2$, and have the other tell which form the equation is in (standard, slope-intercept, or point-slope). Then have students reverse roles. If time permits, reverse the exercise, so that the first student states a form and the second student gives an equation in that form.

B **$(1, -4)$ and $(3, 2)$ are on the line.**

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3$$

Step 2 Substitute the slope and one of the points into the point-slope form. Then write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$ *Use (3, 2).*

$y - 2 = 3x - 9$ *Distribute 3 on the right side.*

$y = 3x - 7$ *Add 2 to both sides.*

C ***x*-intercept = –2, *y*-intercept = 4**

Step 1 Use the intercepts to find two points: $(-2, 0)$ and $(0, 4)$.

Step 2 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$$

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ *Write the slope-intercept form.*

$y = 2x + 4$ *Substitute 2 for m and 4 for b.*

CHECK IT OUT! **Write the equation that describes each line in slope-intercept form.**

3a. slope $= \frac{1}{3}$, $(-3, 1)$ is on the line. $y = \frac{1}{3}x + 2$

3b. $(1, -2)$ and $(3, 10)$ are on the line. $y = 6x - 8$

EXAMPLE 4 Using Two Points to Find Intercepts

The points (4, 8) and (−1, −12) are on a line. Find the intercepts.

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 8}{-1 - 4} = \frac{-20}{-5} = 4$$

Step 2 Write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ *Write the point-slope form.*

$y - 8 = 4(x - 4)$ *Substitute (4, 8) for (x_1, y_1) and 4 for m.*

$y - 8 = 4x - 16$ *Distribute 4 on the right side.*

$y = 4x - 8$ *Add 8 to both sides.*

Step 3 Find the intercepts.

***x*-intercept:**

$y = 4x - 8$ *Replace y with 0 and solve for x.*

$0 = 4x - 8$

$8 = 4x$

$2 = x$

***y*-intercept:**

$y = 4x - 8$ *Use the slope-intercept form to identify the y-intercept.*

$b = -8$

The *x*-intercept is 2, and the *y*-intercept is −8.

CHECK IT OUT! 4. The points (2, 15) and (−4, −3) are on a line. Find the intercepts. ***x*-intercept: −3, *y*-intercept: 9**

Teaching Tip **Communicating Math** Once students have been introduced to multiple methods for graphing a line, discuss with them which method is easiest for different situations.

COMMON ERROR ALERT

When calculating slope, students may put negative numbers in the y_1 and x_1 positions but forget to change subtraction of a negative number to addition. Remind them that subtracting a negative number is the same as adding a positive number.

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Additional Examples

Example 3

Write the equation that describes each line in slope-intercept form.

A. slope $= 3$, $(-1, 4)$ is on the line. $y = 3x + 7$

B. $(2, -3)$ and $(4, 1)$ are on the line. $y = 2x - 7$

C. *x*-intercept $= \frac{1}{4}$, *y*-intercept $= 1$ $y = -4x + 1$

Example 4

The points (10, −3) and (5, −2) are on a line. Find the intercepts. *x*-int.: −5; *y*-int.: −1

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- How do you switch from point-slope form to slope-intercept form?
- Given two points, what are the steps in writing an equation of the line through those points?

EXAMPLE 4

- How do you determine the intercepts once you have written the linear equation in slope-intercept form?

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Additional Examples

Example 5

The cost to stain a deck is a linear function of the deck's area. The costs to stain 100, 250, and 400 square feet are shown in the table. Write an equation in slope-intercept form that represents the function. Then find the cost to stain a deck whose area is 75 square feet.

Area (ft^2)	Cost ($)
100	150
250	337.50
400	525

$y = 1.25x + 25$; \$118.75

Also available online

Questioning Strategies

EXAMPLE 5

- How can you find a point on the line?
- How can you find the slope of the line?

Inclusion Encourage students to look for ways to make problems easier to solve. In **Example 5,** any two points can be used to find the slope, but some pairs may be easier to work with than others.

EXAMPLE 5 *Problem-Solving Application*

Make sense of problems and persevere in solving them.

The cost to place an ad in a newspaper for one week is a linear function of the number of lines in the ad. The costs for 3, 5, and 10 lines are shown. Write an equation in slope-intercept form that represents the function. Then find the cost of an ad that is 18 lines long.

City Gazette

Newspaper Ad Costs

Lines	3	5	10
Cost ($)	13.50	18.50	31

1 Understand the Problem

- The **answer** will have two parts—an equation in slope-intercept form and the cost of an ad that is 18 lines long.
- The ordered pairs given in the table satisfy the equation.

2 Make a Plan

First, find the slope. Then use point-slope form to write the equation. Finally, write the equation in slope-intercept form.

3 Solve

Step 1 Choose any two ordered pairs from the table to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18.50 - 13.50}{5 - 3} = \frac{5}{2} = 2.5$$ *Use (3, 13.50) and (5, 18.50).*

Step 2 Substitute the slope and any ordered pair from the table into the point-slope form.

$y - y_1 = m(x - x_1)$

$y - 31 = 2.5(x - 10)$ *Use (10, 31).*

Step 3 Write the equation in slope-intercept form by solving for y.

$y - 31 = 2.5(x - 10)$

$y - 31 = 2.5x - 25$ *Distribute 2.5.*

$y = 2.5x + 6$ *Add 31 to both sides.*

Step 4 Find the cost of an ad containing 18 lines by substituting 18 for x.

$y = 2.5x + 6$

$y = 2.5(18) + 6 = 51$

The cost of an ad containing 18 lines is \$51.

4 Look Back

Check the equation by substituting the ordered pairs (3, 13.50) and (5, 18.50).

$y = 2.5x + 6$	
13.50	$2.5(3) + 6$
13.5	$7.5 + 6$
13.5	13.5 ✓

$y = 2.5x + 6$	
18.50	$2.5(5) + 6$
18.5	$12.5 + 6$
18.5	18.5 ✓

CHECK IT OUT! **5. What if...?** At a different newspaper, the costs to place an ad for one week are shown. Write an equation in slope-intercept form that represents this linear function. Then find the cost of an ad that is 21 lines long.

Lines	Cost ($)
3	12.75
5	17.25
10	28.50

$y = 2.25x + 6$; \$53.25

3 Close

Summarize

Ask students what they would need to do *first*, given the following information, before writing an equation in slope-intercept form.

slope and (0, 3)	Use the point (0, 3) to identify the y-intercept.
slope and (2, 2)	Write the equation in point-slope form.
2 points on the line	Find the slope.
point-slope form	Simplify.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

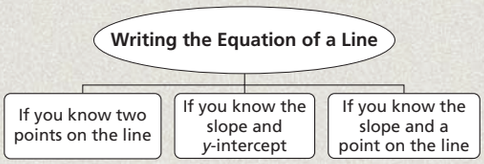
***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. How are point-slope form and slope-intercept form alike? different?
2. When is point-slope form useful? When is slope-intercept form useful?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find the equation of a line by using the given method.

Know it! Note



4-7 Exercises

Learn It Online
Homework Help Online
Parent Resources Online

GUIDED PRACTICE

SEE EXAMPLE 1

Write an equation in point-slope form for the line with the given slope that contains the given point. $y + 6 = \frac{1}{5}(x - 2)$

1. slope $= \frac{1}{5}$; $(2, -6)$
2. slope $= -4$; $(1, 5)$ $y - 5 = -4(x - 1)$
3. slope $= 0$; $(3, -7)$ $y + 7 = 0(x - 3)$

SEE EXAMPLE 2

Graph the line described by each equation.

4. $y - 1 = -(x - 3)$
5. $y + 2 = -2(x + 4)$
6. $y + 1 = -\frac{1}{2}(x + 4)$

SEE EXAMPLE 3

Write the equation that describes each line in slope-intercept form.

7. slope $= -\frac{1}{3}$, $(-3, 8)$ is on the line.
8. slope $= 2$; $(1, 1)$ is on the line.
9. $(-2, 2)$ and $(2, -2)$ are on the line.
10. $(1, 1)$ and $(-5, 3)$ are on the line.
11. x-intercept $= 8$, y-intercept $= 4$
12. x-intercept $= -2$, y-intercept $= 3$

SEE EXAMPLE 4

Each pair of points is on a line. Find the intercepts.

13. $(5, 2)$ and $(7, 4)$ x-int.: 3; y-int.: −3
14. $(-1, 5)$ and $(-3, -5)$ x-int.: −2; y-int.: 10
15. $(2, 9)$ and $(-4, -9)$ x-int.: −1; y-int.: 3

SEE EXAMPLE 5

16. **Measurement** An oil tank is being filled at a constant rate. The depth of the oil is a function of the number of minutes the tank has been filling, as shown in the table. Write an equation in slope-intercept form that represents this linear function. Then find the depth of the oil after one-half hour. $y = \frac{1}{5}x + 3$; 9 ft

Time (min)	Depth (ft)
0	3
10	5
15	6

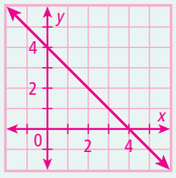
PRACTICE AND PROBLEM SOLVING

Write an equation in point-slope form for the line with the given slope that contains the given point.

17. slope $= \frac{2}{9}$; $(-1, 5)$ 17. $y - 5 = \frac{2}{9}(x + 1)$
18. slope $= 0$; $(4, -2)$ $y + 2 = 0(x - 4)$
19. slope $= 8$; $(1, 8)$ $y - 8 = 8(x - 1)$

Answers

4.



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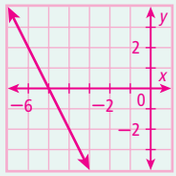

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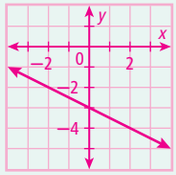


7. $y = -\frac{1}{3}x + 7$
8. $y = 2x - 1$
9. $y = -x$
10. $y = -\frac{1}{3}x + \frac{4}{3}$
11. $y = -\frac{1}{2}x + 4$
12. $y = \frac{3}{2}x + 3$

Answers to *Think and Discuss*

1. Both are based on the slope and a point. Slope-int.: uses the point that contains the y-int.; point-slope: can use any point.
2. Point slope: when you know the slope and a point or 2 points; slope-int.: when you know the slope and the y-int.
3. See Additional Answers.

4-7 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples 1–3

Basic	17–30, 37–39, 44–49
Average	17–30, 37–39, 44–48
Advanced	17–30, 37–39, 44–48, 56–58

If you finished Examples 1–5

Basic	17–39, 43–49, 52–55
Average	17–39, 43–48, 50–56
Advanced	17–48, 50–58

Homework Quick Check
Quickly check key concepts.
Exercises: 18, 20, 24, 32, 34, 36

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 16, 34–36, 43, 52–53

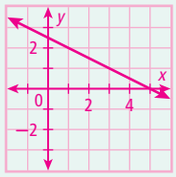
Reason abstractly and quantitatively. Exercises 40–42, 56–58

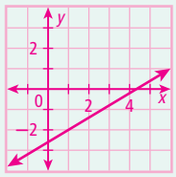
Construct viable arguments and critique the reasoning of others. Exercises 49–51

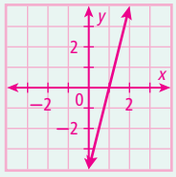
Learn It Online
State Resources Online

Teaching Tip **Science Link Exercise 35** involves finding the boiling point of water in degrees Fahrenheit. Students can check their equation by substituting the altitude at sea level (0) into the equation and checking that the boiling temperature is 212°F.

Answers

20. 

21. 

22. 

23. $y = -\frac{2}{7}x + 1$
24. $y = \frac{4}{5}x + 13$
25. $y = -6x + 57$
26. $y = \frac{1}{7}x + 7$
27. $y = -\frac{11}{2}x + 18$
28. $y = -5x - 3$
29. $y = 2x - 6$
30. $y = \frac{1}{4}x - 1$
31. x-int.: 1; y-int.: −2
32. x-int.: 6; y-int.: 8
33. x-int.: −6; y-int.: 9
35. $y = -\frac{1}{500}x + 212$; 200 °F
36b. −0.95; the change in the amount in dollars remaining on the card after each download
c. 20; the initial amount in dollars on the card

Independent Practice

For Exercises	See Example
17–19	1
20–22	2
23–30	3
31–33	4
34	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Graph the line described by each equation.

20. $y - 4 = -\frac{1}{2}(x + 3)$ **21.** $y + 2 = \frac{3}{5}(x - 1)$ **22.** $y - 0 = 4(x - 1)$

Write the equation that describes each line in slope-intercept form.

23. slope $= -\frac{2}{7}$, (14, −3) is on the line.
24. slope $= \frac{4}{5}$, (−15, 1) is on the line.
25. slope = −6, (9, 3) is on the line.
26. (7, 8) and (−7, 6) are on the line.
27. (2, 7) and (4, −4) are on the line.
28. (−1, 2) and (4, −23) are on the line.
29. x-intercept = 3, y-intercept = −6
30. x-intercept = 4, y-intercept = −1

Each pair of points is on a line. Find the intercepts.

31. (−1, −4) and (6, 10) **32.** (3, 4) and (−6, 16) **33.** (4, 15) and (−2, 6)

34. History The amount of fresh water left in the tanks of a 19th-century clipper ship is a linear function of the time since the ship left port, as shown in the table. Write an equation in slope-intercept form that represents the function. Then find the amount of water that will be left in the ship's tanks 50 days after leaving port.

Fresh Water Aboard Ship	
Time (days)	Amount (gal)
1	3555
8	3240
15	2925

$y = -45x + 3600$; 1350 gal

Science Link

As altitude increases, the amount of breathable oxygen decreases. At elevations above 8000 feet, this can cause altitude sickness. To prevent this, mountain climbers often use tanks containing a mixture of air and pure oxygen.

35. Science At higher altitudes, water boils at lower temperatures. This relationship between altitude and boiling point is linear. At an altitude of 1000 feet, water boils at 210 °F. At an altitude of 3000 feet, water boils at 206 °F. Write an equation in slope-intercept form that represents this linear function. Then find the boiling point at 6000 feet.

36. Consumer Economics Lora has a gift card from an online music store where all downloads cost the same amount. After downloading 2 songs, the balance on her card was $18.10. After downloading a total of 5 songs, the balance was $15.25.

a. Write an equation in slope-intercept form that represents the amount in dollars remaining on the card as a function of songs downloaded. $y = -0.95x + 20$
b. Identify the slope of the line and tell what the slope represents.
c. Identify the y-intercept of the line and tell what it represents.
d. How many additional songs can Lora download when there is $15.25 left on the card? 16 songs

Graph the line with the given slope that contains the given point.

37. slope = −3; (2, 4) **38.** slope $= -\frac{1}{4}$; (0, 0) **39.** slope $= \frac{1}{2}$; (−2, −1)

Tell whether each statement is sometimes, always, or never true.

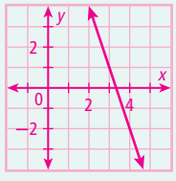
40. A line described by the equation $y = mx + b$ contains the point $(0, b)$. always
41. The slope of the line that contains the points (0, 0) and (c, d) is negative if both c and d are negative. never
42. The y-intercept of the graph of $y - y_1 = m(x - x_1)$ is negative if y_1 is negative. sometimes

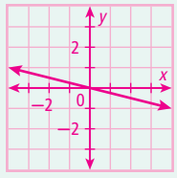
43. Meteorology Snowfall accumulates at an average rate of 2.5 inches per hour during a snowstorm. Two hours after the snowstorm begins, the average depth of snow on the ground is 11 inches.

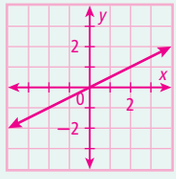
a. Write an equation in point-slope form that represents the depth of the snow in inches as a function of hours since the snowstorm began. $y - 11 = 2.5(x - 2)$
b. How much snow is on the ground when the snowstorm starts? 6 in.
c. The snowstorm begins at 2:15 P.M. and continues until 6:30 P.M. How much snow is on the ground at the end of the storm? $16\frac{5}{8}$ in.

© Royal Geographical Society/Alamy

Answers

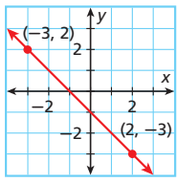
37. 

38. 

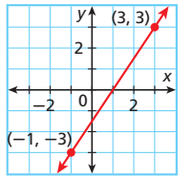
39. 

Write an equation in point-slope form that describes each graph.

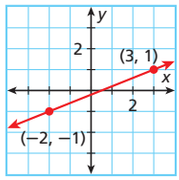
44. Possible answer: $y + 3 = -(x - 2)$



45. Possible answer: $y - 3 = \frac{3}{2}(x - 3)$



46. Possible answer: $y - 1 = \frac{2}{5}(x - 3)$



The tables show linear relationships between x and y. Copy and complete the tables.

47.

x	−2	0	4	7
y	−18	−8	12	27

48.

x	−4	1	0	6
y	14	4	6	−6

49. **ERROR ANALYSIS** Two students used point-slope form to find an equation that describes the line with slope −3 through $(-5, 2)$. Who is incorrect? Explain the error.

A

$y - y_1 = m(x - x_1)$

$y - 2 = -3(x - 5)$

B

$y - y_1 = m(x - x_1)$

$y - 2 = -3[x - (-5)]$

$y - 2 = -3(x + 5)$

Student A is incorrect. Student A incorrectly wrote $x - (-5)$ as $x - 5$ instead of $x + 5$.

50. **Critical Thinking** Compare the methods for finding the equation that describes a line when you know

- a point on the line and the slope of the line.
- two points on the line.

How are the methods alike? How are they different?

50. Possible answer: When you know a point and the slope, you can immediately use point-slope form. When you know 2 points, first use them to find the slope. Then use point-slope form, just like in the first case.

51. **Write About It** Explain why the first statement is false but the second is true.

- All linear equations can be written in point-slope form.
- All linear equations that describe functions can be written in point-slope form.

51. Possible answer: Linear equations that describe vert. lines cannot be written in point-slope form because they have undef. slope. All non-vert. lines represent functions, and they can all be written in point-slope form.

52. **Multi-Step** The table shows the mean scores on a standardized test for several different years.

Years Since 1985	0	5	10	17	21
Mean Combined Score	994	1009	1001	1016	1020

a. Make a scatter plot of the data and add a trend line to your graph.

b. Use your trend line to estimate the slope and y-intercept, and write an equation in slope-intercept form. **Possible answer: slope: 1.5; y-int.: 994; $y = 1.5x + 994$**

c. What do the slope and y-intercept represent in this situation? **slope: number of points by which mean score is increasing each year; y-int.: mean score in 1985**

MULTI-STEP TEST PREP

© Brand X Pictures

53. **a.** Stephen is walking from his house to his friend Sharon's house. When he is 12 blocks away, he looks at his watch. He looks again when he is 8 blocks away and finds that 6 minutes have passed. Write two ordered pairs for these data in the form (time, blocks). **(0, 12) and (6, 8)**

b. Write a linear equation for these two points. $y = -\frac{2}{3}x + 12$

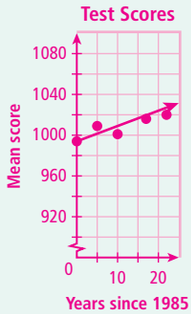
c. What is the total amount of time it takes Stephen to reach Sharon's house? Explain how you found your answer. **18 min**

Teaching Tip **Statistics** In **Exercise 52,** remind students that a trend line should have about the same number of points below it as above it.

MULTI-STEP TEST PREP **Exercise 53** involves using two points to write an equation for a line. This exercise prepares students for the Multi-Step Test Prep.

Answers

52a.



READING STRATEGIES

The point-slope form of a line can be used when writing linear equations.

Equation

$y - y_1 = m(x - x_1)$

Point-Slope Form

Writing Equations in Point-Slope Form
A line with slope 3 which contains the point (–6, 2) can be written in point-slope form as $y - 2 = 3[x - (-6)]$, or $y - 2 = 3(x + 6)$.

Changing Point-Slope Form to Slope-Intercept Form
$y - 2 = 3(x + 6)$
$y - 2 = 3x + 18$
$y = 3x + 20$

Uses
The point-slope form can be used to write a linear equation when given either...
- a point and the slope
- two points

...and then simplified to slope-intercept form to graph.

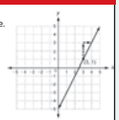
Complete the following.

1. What two pieces of information do you need to write an equation in point-slope form?
   the slope of the line and one point on the line
2. In the concept map above, it states that you can write an equation in point-slope form if given two points on the line. What piece of information do you still need and how can you get it?
   the slope; you can substitute the coordinates of the two given points into the slope formula.
3. Write an equation in point-slope form for the line with slope 4 that contains (3, –10).
   $y + 10 = 4(x - 3)$
4. Write an equation in slope-intercept form for the line that contains points (2, –5) and (–4, 7).
   $y = -2x - 1$

RETEACH

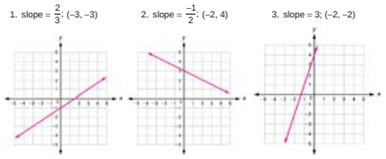
You can graph a line if you know the slope and any point on the line.

Graph the line with slope 2 that contains the point (3, 1).
Step 1: Plot (3, 1).
Step 2: The slope is 2 or $\frac{2}{1}$. Count **2 up** and **1 right** and plot another point.
Step 3: Draw a line connecting the points.

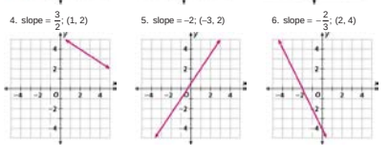


Graph the line with the given slope that contains the given point.

1. slope $= \frac{2}{3}$; (–3, –3)
2. slope $= \frac{-1}{2}$; (–2, 4)
3. slope = 3; (–2, –2)



4. slope $= \frac{3}{2}$; (1, 2)
5. slope = –2; (–3, 2)
6. slope $= -\frac{2}{3}$; (2, 4)



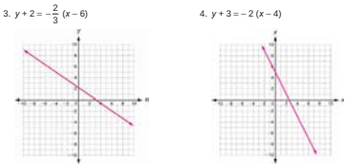
PRACTICE A
PRACTICE C
PRACTICE B

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. slope = 3; (–4, 2) — **$y - 2 = 3(x + 4)$**
2. slope = –1; (6, –1) — **$y + 1 = -(x - 6)$**

Graph the line described by each equation.

3. $y + 2 = -\frac{2}{3}(x - 6)$
4. $y + 3 = -2(x - 4)$



Write the equation that describes the line in slope-intercept form.

5. slope = –4; (1, –3) is on the line — **$y = -4x + 1$**
6. slope $= \frac{1}{2}$; (–8, –5) is on the line — **$y = \frac{1}{2}x - 1$**
7. (2, 1) and (0, –7) are on the line — **$y = 4x - 7$**
8. (–6, –6) and (2, –2) are on the line — **$y = \frac{1}{2}x - 3$**

Find the intercepts of the line that contains each pair of points.

9. (–1, –4) and (6, 10) **x-int: 1, y-int: –2**
10. (3, 4) and (–6, 16) **x-int:6, y-int: 8**
11. The cost of internet access at a cafe is a function of time. The costs for 8, 25, and 40 minutes are shown. Write an equation in slope-intercept form that represents the function. Then find the cost of surfing the web at the cafe for one hour. **$y = 0.17x + 3$; \$13.20**

Time (min)	8	25	40
Cost ($)	4.36	7.25	9.80

TEST PREP DOCTOR In **Exercise 54,** students who chose **B** most likely reversed x_1 and y_1 in the point-slope form. Students who chose **A** probably added x_1 and y_1.

Journal

Have students describe a situation in which the point-slope form is more useful than the slope-intercept form.

ALTERNATIVE ASSESSMENT

Have students choose two points on a coordinate grid and write an equation in slope-intercept form for the line that passes through the points. Then have them graph the line and check their work by making sure their two original points are on the line.

Power Presentations with PowerPoint®

4-7 Lesson Quiz

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. slope $= -1$; $(0, 9)$
$y - 9 = -(x - 0)$

2. slope $= -\frac{1}{3}$; $(3, -6)$
$y + 6 = -\frac{1}{3}(x - 3)$

Write the equation that describe each line in slope-intercept form.

3. slope $= -2$, $(2, 1)$ is on the line.
$y = -2x + 5$

4. $(0, 4)$ and $(-7, 2)$ are on the line.
$y = \frac{2}{7}x + 4$

5. The cost to take a taxi from the airport is a linear function of the distance driven. The costs for 5, 10, and 20 miles are shown in the table. Write an equation in slope-intercept form that represents the function.

Distance (mi)	Cost ($)
5	14
10	22
20	38

$y = 1.6x + 6$

Also available online

TEST PREP

54. Which equation describes the line through $(-5, 1)$ with slope of 1?
(A) $y + 1 = x - 5$ (C) $y - 1 = -5(x - 1)$
(B) $y + 5 = x - 1$ (D) $y - 1 = x + 5$

55. A line contains (4, 4) and (5, 2). What are the slope and *y*-intercept?
(F) slope $= -2$; *y*-intercept $= 2$ (H) slope $= -2$; *y*-intercept $= 12$
(G) slope $= 1.2$; *y*-intercept $= -2$ (J) slope $= 12$; *y*-intercept $= 1.2$

CHALLENGE AND EXTEND

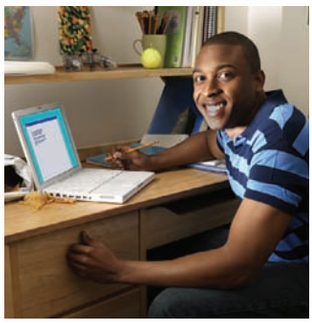
56. A linear function has the same *y*-intercept as $x + 4y = 8$ and its graph contains the point $(2, 7)$. Find the slope and *y*-intercept. **slope: $\frac{5}{2}$; *y*-int.: 2**

57. Write the equation of a line in slope-intercept form that contains $\left(\frac{3}{4}, \frac{1}{2}\right)$ and has the same slope as the line described by $y + 3x = 6$. **$y = -3x + \frac{11}{4}$**

58. Write the equation of a line in slope-intercept form that contains $\left(-\frac{1}{2}, -\frac{1}{3}\right)$ and $\left(1\frac{1}{2}, 1\right)$. **$y = \frac{2}{3}x$**

Career Path

Learn It Online
Career Resources Online



Michael Raynor
Data mining major

Q: What math classes did you take in high school?
A: Algebra 1 and 2, Geometry, and Statistics

Q: What math classes have you taken in college?
A: Applied Statistics, Data Mining Methods, Web Mining, and Artificial Intelligence

Q: How do you use math?
A: Once for a class, I used software to analyze basketball statistics. What I learned helped me develop strategies for our school team.

Q: What are your future plans?
A: There are many options for people with data mining skills. I could work in banking, pharmaceuticals, or even the military. But my dream job is to develop game strategies for an NBA team.

© Ocean/Corbis

PROBLEM SOLVING

Write the correct answer.

1. The number of students in a school has been increasing at a constant rate. The table shows the number of students in the school for certain numbers of years since 1995.

Years Since 1995	Number of Students
0	118
5	124
10	130

Write an equation in point-slope form that represents this linear function.
Possible answer: $y - 130 = 1.2(x - 10)$
Write the equation in slope-intercept form.
$y = 1.2x + 118$
Assuming the rate of change remains constant, how many students will be in the school in 2010?
136

2. Toni is finishing a scarf at a constant rate. The table shows the number of hours Toni has spent knitting this week and the corresponding number of rows in the scarf.

Toni's Knitting	
Hours	Rows of Knitting
2	38
4	44
6	50

Write an equation in slope-intercept form that represents this linear function.
$y = 3x + 32$

3. A photo lab manager graphed the cost of having photos developed as a function of the number of photos in the order. The graph is a line with a slope of $\frac{1}{10}$ that passes through (10, 6). Write an equation in slope-intercept form that describes the cost to have photos developed. How much does it cost to have 25 photos developed?
$y = \frac{1}{10}x + 5$; $7.50

The cost of a cell phone for one month is a linear function of the number of minutes used. The total cost for 20, 35, and 40 additional minutes are shown. Select the best answer.

Cell-Phone Costs			
Number of Additional Minutes	20	35	40
Total Cost	$48	$54	$56

4. What is the slope of the line represented in the table?
A 0.1 C 2
B 0.4 D 2.5

5. What would be the monthly cost if 60 additional minutes were used?
F $64 H $84
G $72 J $150

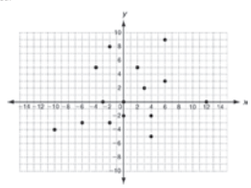
6. What does the *y*-intercept of the function represent?
A total cost of the bill
B cost per additional minute
C number of additional minutes used
D cost with no additional minutes used

CHALLENGE

The objective of this game is to write and graph linear equations such that each line passes through as many points as possible. Each equation must be written in slope-intercept form, but you may use point-slope form as part of your work.

Two-player version:
- Each player takes a turn writing and graphing one linear equation that passes through points on the graph. Allow time for the player to make calculations. Check each other's work for accuracy—sometimes a line may come close to a point but not actually intersect it.
- The player scores 5 points for each point on the graph that the line passes through. If the line passes through a point that was previously intersected by a line, the player does not get credit for it.
- After all points have been used, tally your scores to see who wins.

Single-player version:
- Try to write equations for the fewest number of lines that intersect all of the points.
- If other students in your class are playing as single players, compare results to see who was able to use the fewest lines. Challenge yourselves to find ways to further reduce the number of lines that you used.



Tell how to make a game board that would result in a tie.
Plot an even number of pairs of points such that the two points in each pair are collinear but neither point is collinear with any other plotted point.

4-7 Technology LAB

Graph Linear Functions

You can use a graphing calculator to quickly graph lines whose equations are in point-slope form. To enter an equation into your calculator, it must be solved for y, but it does not necessarily have to be in slope-intercept form.

Use with Point-Slope Form

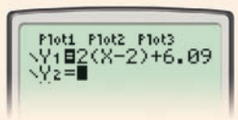
Activity

Graph the line with slope 2 that contains the point $(2, 6.09)$.

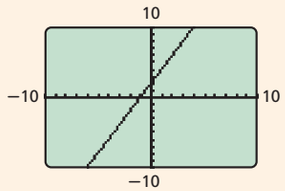
1. Use point-slope form.
   $y - y_1 = m(x - x_1)$
   $y - 6.09 = 2(x - 2)$

2. Solve for y by adding 6.09 to both sides of the equation.
   $y - 6.09 = 2(x - 2)$
   $+6.09 \quad +6.09$
   $y = 2(x - 2) + 6.09$

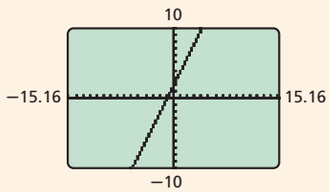
3. Enter this equation into your calculator.
   Y= 2 (X,T,θ,n) − 2) + 6.09 ENTER



4. Graph in the *standard viewing window* by pressing ZOOM and selecting **6:ZStandard**. In this window, both the x- and y-axes go from -10 to 10.



5. Notice that the scale on the y-axis is smaller than the scale on the x-axis. This is because the width of the calculator screen is about 50% greater than its height. To see a more accurate graph of this line, use the *square viewing window*. Press ZOOM and select **5:ZSquare**.



Try This

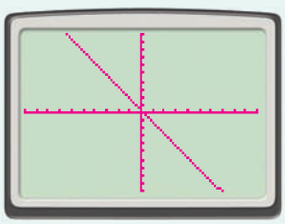
1. Graph the function represented by the line with slope -1.5 that contains the point $(2.25, -3)$. View the graph in the standard viewing window.
2. Now view the graph in the square viewing window. Press WINDOW and write down the minimum and maximum values on the x- and y-axes.
3. In which graph does the line appear steeper? Why?
4. Explain why it might sometimes be useful to look at a graph in a square window.

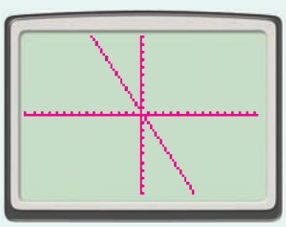
Learn It Online
Lab Resources Online

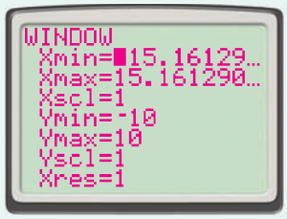
MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

Answers

1. 

2. 

3. 

The second graph; the stand. window has the same number of units on each axis. Since the calc. screen is more horiz. than vert., the units on the x-axis are larger than the units on the y-axis. This makes the line appear less steep than it really is. In the square window, the units on each axis are the same size, so the slope of the line is represented more accurately.

4. Possible answer: When you want an accurate representation of the slope of a line, a square window would help avoid skewing the line.

Technology LAB Organizer

Use with the lesson Point-Slope Form

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to graph lines.

Materials: graphing calculator

INTERACTIVE **Online Edition**
Graphing Calculator, TechKeys

Countdown

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

Have students compare the graph in the two viewing windows. Discuss how the scale of the viewing window can distort a graph.

Close

Key Concept

Before graphing a function on a calculator, you must first solve for y. The scale of the axes in the viewing window affects the appearance of the slope of a line.

Assessment

Journal Have students explain the difference between the standard viewing window and the square viewing window.

Teaching Tip **Technology** Caution students not to confuse the negative key (−) with the subtraction key −.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph…*
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Learn It Online
State Resources Online

Organizer

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Write and interpret a linear equation for a trend line on a scatter plot.

Online Edition

Teach

Remember

Students review the slope formula, the definition of *y*-intercept, and the slope-intercept form of a linear equation.

Discuss

Be sure students understand that the equation of a trend line cannot be used to make predictions with absolute certainty. Instead, it is used to make a "best guess" based on the available data.

Close

Assess

Use the graph from the example. Have students describe another method, different from the one shown, to find the equation of the trend line. Possible answer: After finding the slope, substitute that value and one of the ordered pairs on the trend line into the point-slope form.

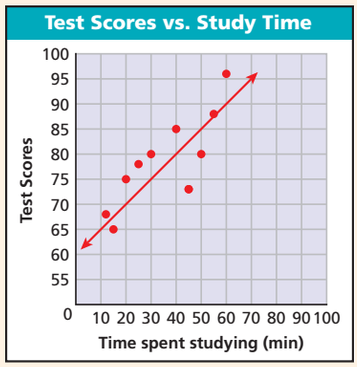
Learn It Online
State Resources Online

Interpreting Trend Lines

Previously, you learned how to draw trend lines on scatter plots. Now you will learn how to find the equations of trend lines and write them in slope-intercept form.

Example

Write an equation for the trend line on the scatter plot.



Two points on the trend line are (30, 75) and (60, 90).

To find the slope of the line that contains (30, 75) and (60, 90), use the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$m = \frac{90 - 75}{60 - 30}$ *Substitute (30, 75) for (x_1, y_1) and (60, 90) for (x_2, y_2).*

$m = \frac{15}{30}$ *Simplify.*

$m = \frac{1}{2}$

Use the slope and the point (30, 75) to find the *y*-intercept of the line.

$y = mx + b$ *Slope-intercept form*

$75 = \frac{1}{2}(30) + b$ *Substitute $\frac{1}{2}$ for m, 30 for x, and 75 for y.*

$75 = 15 + b$ *Solve for b.*

$60 = b$

Write the equation.

$y = \frac{1}{2}x + 60$ *Substitute $\frac{1}{2}$ for m and 60 for b.*

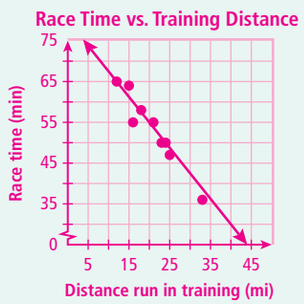
Try This

1. In the example above, what is the meaning of the slope? **Every two minutes spent studying results in a one-point increase in the test score.**
2. What does the *y*-intercept represent? **The score of a student who spent no time studying**
3. Use the equation to predict the test score of a student who spent 25 minutes studying. **72.5**
4. Use the table to create a scatter plot. Draw a trend line and find the equation of your trend line. Tell the meaning of the slope and *y*-intercept. Then use your equation to predict the race time of a runner who ran 40 miles in training.

Distance Run in Training (mi)	12	15	16	18	21	23	24	25	33
Race Time (min)	65	64	55	58	55	50	50	47	36

Answers to *Try This*

4. Possible answers:



$y = -\frac{5}{4}x + 80$; the slope, $-\frac{5}{4}$, indicates that every 4 miles run in training decreases the race time by 5 minutes; the *y*-intercept, 80, indicates the race time of someone who did not run any distance in training; 30 min

COMMON CORE

4-8 Line of Best Fit

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *Also* **CC.9-12.S.ID.6b, CC.9-12.S.ID.7, CC.9-12.S.ID.8, CC.9-12.S.ID.9**

Objectives
Determine a line of best fit for a set of linear data.

Determine and interpret the correlation coefficient.

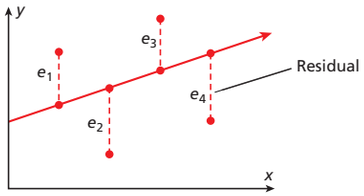
Vocabulary
residual
least-squares line
line of best fit
linear regression
correlation coefficient

Who uses this?

Climate scientists can use a least-squares line to study temperature-latitude relationships. (See Example 2.)

Recall that a scatter plot shows two data sets as one set of ordered pairs. A trend line, or line of fit, is a model for the data.

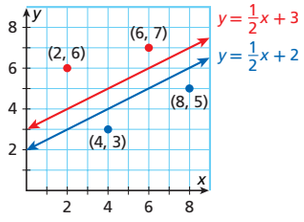
Some trend lines will fit a data set better than others. One way to evaluate how well a line fits a data set is to use *residuals*. A **residual** is the signed vertical distance between a data point and a line of fit. The closer the sum of the squared residuals is to 0, the better the line fits the data.



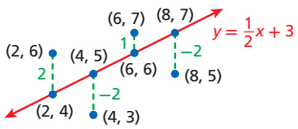
EXAMPLE 1 Calculating Residuals

The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit?

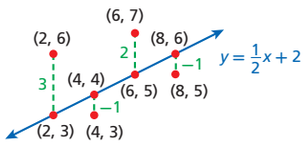
x	2	4	6	8
y	6	3	7	5



Find the residuals.



Sum of squared residuals:
$(2)^2 + (-2)^2 + (1)^2 + (-2)^2$
$4 + 4 + 1 + 4 = 13$



Sum of squared residuals:
$(3)^2 + (-1)^2 + (2)^2 + (-1)^2$
$9 + 1 + 4 + 1 = 15$

The line $y = \frac{1}{2}x + 3$ is a better fit for the data.

Helpful Hint

By using *squares* of residuals, positive and negative residuals do not "cancel out," and residuals with squares greater than 1 have a magnified effect on the sum.

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4-8 Organizer

Pacing: Traditional 2 days
Block 1 day

Objectives: Determine a line of best fit for a set of linear data.

Determine and interpret the correlation coefficient.

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Warm Up

Identify the slope and the *y*-intercept.

1. $y = -2x + 1$ $m = -2, b = 1$
2. $y = \frac{2}{3}x - 4$ $m = \frac{2}{3}, b = -4$

Identify the correlation (positive, negative, or none) that you would expect to see between each pair of data sets.

3. a person's height and shoe size pos.
4. the age of a car and its value neg.

Also available online

Math Humor

Q: Why did the playwright take so long to decide what the lead character should say at a crucial moment in the play?

A: She wanted to be sure it was the line of best fit.

COMMON CORE

CC.9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
b. Informally assess the fit of a function by plotting and analyzing residuals.
CC.9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
CC.9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
CC.9-12.S.ID.9 Distinguish between correlation and causation.

Learn It Online
State Resources Online

1 Introduce

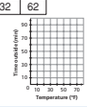
EXPLORATION

Line of Best Fit

The table shows the temperatures for five consecutive days and the number of minutes Rosa spent outside that day.

Temperature (°F)	77	70	72	51	60
Time Outside (min)	85	95	91	32	62

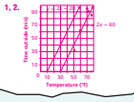
1. Plot the ordered pairs on the grid at right.
2. Graph and label the trend lines represented by $y = 2x - 20$ and $y = 2x - 60$ on the grid.



THINK AND DISCUSS

3. **Describe** how to use each equation to predict the amount of time Rosa will spend outside on a day when the temperature is 80 °F.
Substitute 80 for *x* in each equation and find the value of *y*.
$y = 2x - 20 = 2(80) - 20 = 140$ min
$y = 2x - 60 = 2(80) - 60 = 100$ min
4. **Explain** which of the predictions you think is more accurate and why.
Possible answer: 100 min; $y = 2x - 60$ is closer to the data points than $y = 2x - 20$.

1, 2.



Motivate

Previously, students learned that a scatter plot can show a correlation between two data sets, and that a trend line can help to show that correlation. On the board, make a rough sketch of a scatter plot. Then sketch two trend lines and label them ℓ_1 and ℓ_2. Ask students, by a show of hands, to indicate which line they think better fits the data. Then tell them that they will learn how to find a line of *best* fit for a data set.

Explorations and answers are provided in your online resources.

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Additional Examples

Example 1

Two lines of fit for this data are $y = 2x + 2$ and $y = x + 4$. For each line, find the sum of the squares of the residuals. Which line is a better fit?

x	1	2	3	4
y	7	5	6	9

$y = 2x + 2$: 15; $y = x + 4$: 7; $y = x + 4$ is a better fit.

Example 2

The table shows populations and numbers of U.S. Representatives for several states in the year 2000.

State	Population (millions)	Representatives
AL	4.5	7
AK	0.6	1
AZ	5.1	8
AR	2.7	4
CA	33.9	53
CO	4.3	7

A. Find an equation for a line of best fit. $y = 1.56x + 0.02$

B. Interpret the meaning of the slope and y-intercept. Slope: for each 1 million increase in pop., the number of Representatives increases by 1.56 million; y-int.: a state with a pop. of 0 (or less than a million) has 0.02 Representatives (or 1 Representative).

C. Michigan had a population of approximately 10.0 million in 2000. Use your equation to predict Michigan's number of Representatives. 16

Also available online

CHECK IT OUT! **1.** Two lines of fit for this data are $y = -\frac{1}{2}x + 6$ and $y = -x + 8$. For each line, find the sum of the squares of the residuals. Which line is a better fit?

x	2	4	6	8
y	3	6	1	4

$y = -\frac{1}{2}x + 6$: 16; $y = -x + 8$: 30; $y = -\frac{1}{2}x + 6$ is better.

The **least-squares line** for a data set is the line of fit for which the sum of the squares of the residuals is as small as possible. So, the least-squares line is a *line of best fit*. A **line of best fit** is the line that comes closest to all of the points in the data set, using a given process. **Linear regression** is a process of finding the least-squares line.

EXAMPLE 2 Finding the Least-Squares Line

The table shows the latitudes and average temperatures of several cities.

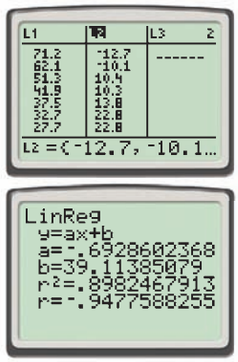
City	Latitude	Average Temperature (°C)
Barrow, Alaska, USA	71.2° N	−12.7
Yakutsk, Russia	62.1° N	−10.1
London, England	51.3° N	10.4
Chicago, Illinois, USA	41.9° N	10.3
San Francisco, California, USA	37.5° N	13.8
Yuma, Arizona, USA	32.7° N	22.8
Tindouf, Algeria	27.7° N	22.8
Dakar, Senegal	14.0° N	24.5
Mangalore, India	12.5° N	27.1

A **Find an equation for a line of best fit.**

Use your calculator. To enter the data, press STAT and select **1:Edit**. Enter the latitudes in the **L1** column and the average temperatures in the **L2** column.

Then press STAT and choose **CALC**.

Choose **4:LinReg(ax+b)** and press ENTER. An equation for a line of best fit is $y \approx -0.69x + 39.11$.



B **Interpret the meaning of the slope and y-intercept.**

The slope, −0.69, means that for each 1-degree increase in latitude, the average temperature decreases 0.69 °C. The y-intercept, 39.11, means that the average temperature is 39.11 °C at 0° N latitude.

C **The approximate latitude of Vancouver, Canada, is 49.1° N. Use your equation to predict Vancouver's average temperature.**

$y \approx -0.69x + 39.11$

$y \approx -0.69(49.1) + 39.11 \approx 5.23$

The average temperature of Vancouver should be close to 5 °C.

2 Teach

Guided Instruction

Before beginning this lesson, review $y = mx + b$, the slope-intercept form of a linear equation. Remind students that m is the slope and b is the y-intercept. Explain that the trend lines shown in **Example 1** were chosen to roughly follow the pattern in the scatter plot, but neither is a line of *best* fit. When discussing **Examples 2–4**, remind students that each equation *does* represent a line of best fit, and that the equations are obtained by calculator software that compares residuals.

Reaching All Learners

Through Multiple Representations

For **Example 2**, have students write these notes: L1 and x represent Latitude; L2 and y represent Average Temperature. Next, have them set up a coordinate system on graph paper, using these scales: 0 to 75 by 5's on the x-axis and −15 to 40 by 5's on the y-axis. Tell students to graph the data. Next, tell them to graph $y = -0.7x + 40$ (an approximation of $y = -0.69x + 39.11$) and confirm that the line is a good fit for the data.

Questioning Strategies

EXAMPLE 1

- Why are residuals of 2 and −2 equivalent measures of fit?
- How do you use residuals to decide which line is a better fit?

EXAMPLE 2

- How are the variables x and y related to L1 and L2?

CHECK IT OUT! **2.** The table shows the prices and the lengths in yards of several balls of yarn at Knit Mart.

Length (yd)	1680	100	153	99	109	109	176	100	1440	61
Price ($)	65.85	7.85	9.80	10.85	8.35	7.85	19.85	5.35	65.85	14.85

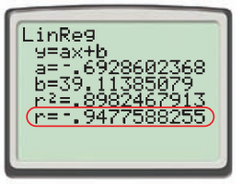
a. Find an equation for a line of best fit. $y \approx 0.04x + 6.38$

b. Interpret the meaning of the slope and *y*-intercept.

c. Knit Mart also sells yarn in a 1000-yard ball. Use your equation to predict the cost of this yarn. $46.38

2b. Slope: cost is $0.04/yd; *y*-int.: $6.38 is added to the cost of every ball of yarn.

In Example 2, you may have noticed the last value the calculator gave you, *r*. This is the *correlation coefficient*. The **correlation coefficient** is a number *r*, where $-1 \le r \le 1$, that describes how closely the points in a scatter plot cluster around a line of best fit.



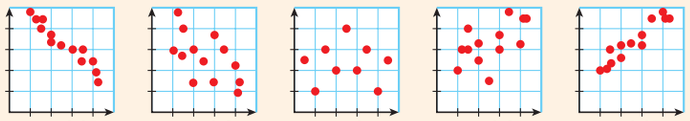
Properties of the Correlation Coefficient *r*

r is a value in the range $-1 \le r \le 1$.

If $r = 1$, the data set forms a straight line with a positive slope.

If $r = 0$, the data set has no correlation.

If $r = -1$, the data set forms a straight line with a negative slope.



Helpful Hint

r-values close to 1 or –1 indicate a very strong correlation. The closer *r* is to 0, the weaker the correlation.

EXAMPLE 3 Correlation Coefficient

The table shows a relationship between a city's population and the average time the city's citizens spend commuting to work each day.

City	Population (thousands)	Average Commute Time (min)
Albuquerque, NM	505	21.5
Atlanta, GA	486	31.1
Austin, TX	710	23.2
Charlotte, NC	630	25.1
Chicago, IL	2833	30.6
Eugene, OR	146	17.9
Houston, TX	2144	27.7
Las Vegas, NV	553	25.2
New York, NY	8496	34.0
New Orleans, LA	223	24.2

COMMON ERROR ALERT

Students may think the value of *r* is related to the steepness of the line of fit. For example, they may think a line of fit with $r = 0.8$ is steeper than a line of fit with $r = 0.6$. Explain that *r* indicates the strength of a correlation, not steepness.

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Additional Examples

Example 3

The table shows a relationship between points allowed and games won by a football team over eight seasons.

Year	Points Allowed	Games Won
1	285	3
2	310	4
3	301	3
4	186	6
5	146	7
6	159	7
7	170	5
8	190	6

Find an equation for a line of best fit. How well does the line represent the data?

$y \approx -0.02x + 9.91$; very well ($r \approx -0.91$)

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 3

- Why does an *r*-value of 0.61 indicate a moderate positive correlation?

3 Close

Summarize

Have students complete the statements.

A residual is the signed vertical distance between a _____ and a _____. data point; line of fit

A line of best fit that is based on minimizing squares of the residuals is called the _____. least-squares line

Correlation coefficients that indicate strong correlations are near the value ___ or ___. 1; –1

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

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Additional Examples

Example 4

Malik is a contractor, installing windows for a builder. The table shows data for his first eight weeks on the job. The equation of the least-squares line for the data is $y \approx -10.36x + 53$, and $r \approx -0.88$. Discuss correlation and causation for the data set.

Week	Average Time per Window (hr)	Net Profit per Hour ($)
1	3.5	19
2	2.8	25
3	2.5	24
4	2.1	26
5	2.3	30
6	1.9	37
7	1.7	35
8	1.8	39

strong neg. corr.; likely cause-and-effect (likely that less installation time contributes to a greater profit per hour)

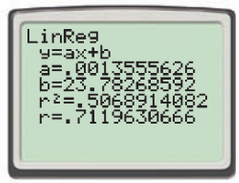
Also available online

Find an equation for a line of best fit. How well does the line represent the data?

Use your calculator.

Enter the data into the lists **L1** and **L2**.

Then press STAT and choose **CALC**. Choose **4:LinReg(ax+b)** and press ENTER. An equation for a line of best fit is $y \approx 0.001x + 23.8$. The value of r is about 0.71, which indicates a moderate positive correlation.



CHECK IT OUT!

3. Kylie and Marcus designed a quiz to measure how much information adults retain after leaving school. The table below shows the quiz scores of several adults, matched with the number of years each person had been out of school. Find an equation for a line of best fit. How well does the line represent the data? **$y \approx -2.74x + 84.32$; very well ($r \approx -0.88$)**

Time Out of School (yr)	1	1	1	2	2	3	5	7	10	10	14	25
Quiz Score	85	94	98	75	80	77	63	56	45	50	34	33

Causation refers to cause-and-effect. If a change in one variable directly causes a change in the other variable, then there is a cause-and-effect relationship between the variables. There is often correlation without causation.

EXAMPLE 4 Correlation and Causation

The table shows test averages of eight students. The equation of the least-squares line for the data is $y \approx 0.77x + 18.12$ and $r \approx 0.87$. Discuss correlation and causation for the data set.

U.S. History Test Average	90	70	75	100	90	85	80	90
Science Test Average	80	75	72	95	92	82	80	92

There is a strong positive correlation between the U.S. history test average and the science test average for these students. There is *not* a likely cause-and-effect relationship because there is no apparent reason why test scores in one subject would directly affect test scores in the other subject.

Caution!

Notice in Example 4, there is a strong correlation, but no causation. Two variables can be strongly correlated without having a direct cause-and-effect relationship.

CHECK IT OUT!

4. Eight adults were surveyed about their education and earnings. The table shows the survey results. The equation of the least-squares line for the data is $y \approx 5.59x - 30.28$ and $r \approx 0.86$. Discuss correlation and causation for the data set.

Years of Education	12	16	20	14	18	16	16	18
Earnings Last Year (thousand $)	40	65	75	44	70	50	54	86

4. strong pos. corr.; likely cause-and-effect (more education often contributes to higher earnings)

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- What is a possible explanation for a positive correlation between history test averages and science test averages?

THINK AND DISCUSS

MATHEMATICAL PRACTICES

1. What is the residual for a data point that lies on the line of best fit?
2. **GET ORGANIZED** Copy and complete the graphic organizer. For each *r*-value, sketch a possible scatter plot and describe the correlation, choosing from the following: strong positive, weak positive, none, strong negative, weak negative.

r-value	−0.9	−0.4	0	0.4	0.9
Scatter Plot					
Description of Correlation					

4-8 Exercises

GUIDED PRACTICE

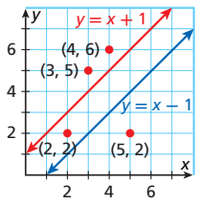
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. A signed vertical distance between a data point and its corresponding model point is called a _____?_____ (*residual* or *correlation coefficient*) **residual**
2. A _____?_____ (*least squares line* or *correlation coefficient*) is a measure of how well a line of best fit models a data set. **correlation coefficient**

SEE EXAMPLE 1

3. The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit for the data?

x	2	3	4	5
y	2	5	6	2



$y = x + 1$: 19; $y = x - 1$: 23; $y = x + 1$ is better.

SEE EXAMPLE 2

4. The table shows numbers of books read by students in an English class over a summer and the students' grades for the following semester.

Books	0	0	0	0	1	1	1	2	3	5	6	8	10	12	20
Grade	65	69	70	73	70	75	78	77	86	85	89	90	95	99	98

   a. Find an equation for a line of best fit. **$y \approx 1.72x + 73.35$**
   b. Interpret the meaning of the slope and *y*-intercept.
   c. Use your equation to predict the grade of a student who reads 15 books. **≈99.15, or 99**

4b. Slope: for each bk. read, student's avg. will increase 1.72 pts; *y*-int.: a student who reads 0 bks. will have an avg. of 73.35.

Answers to *Think and Discuss*

1. 0
2. See Additional Answers.

4-8 Exercises

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–2**
Basic 7, 8, 18, 21–33
Average 7, 8, 12, 14, 18–33
Advanced 7, 8, 12, 14, 18–33

If you finished Examples **1–4**
Basic 7–14, 17, 18, 20
Average 7–20
Advanced 7–20

Homework Quick Check
Quickly check key concepts.
Exercises: 7, 8, 9, 10, 12, 14

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 4–6, 8–10, 13–16, 19

Reason abstractly and quantitatively. Exercises 3, 7

Construct viable arguments and critique the reasoning of others. Exercises 11–12

Look for and make use of structure. Exercise 20

Learn It Online
State Resources Online

Teaching Tip **Visual** Some students may have difficulty comparing the absolute values of decimals in **Exercise 11**. Encourage these students to draw a number line, place the corresponding points in approximately the correct locations, and then determine which is closer to 0.

SEE EXAMPLE 3

5. A negative correlation exists between the time Shawnda spends on homework during an evening and the amount of sleep she gets that night. The table shows data for several nights. Find an equation for a line of best fit. How well does the line represent the data?

Homework (h)	0.5	0.5	1	1	1.5	2	2	2.5	3	3	3	4	4.5	5
Sleep (h)	8	9	8	8.5	8	7.5	8	7.5	7	7	8	6.5	6.5	6

$y \approx -0.53x + 8.8$; very well ($r \approx -0.91$)

SEE EXAMPLE 4

6. Some students were surveyed about how much time they spent playing video games last week and their overall test average. The equation of the least-squares line for the data is $y \approx -2.93x + 89.70$ and $r \approx -0.92$. Discuss correlation and causation for the data set. strong neg. corr.; likely cause-and-effect (more time playing video games often contributes to lower test avgs.)

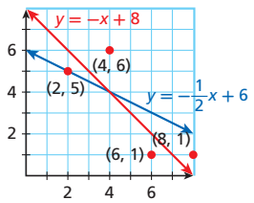
Hours Playing Video Games	1	3	3	6	2	1	9	10
Test Average for all Subjects	80	85	78	70	86	92	60	64

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercise	See Example
7	1
8	2
9	3
10	4

7. The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit for the data?



x	2	4	6	8
y	5	6	1	1

$y = -x + 8$: 7;
$y = -\frac{1}{2}x + 6$: 9;
$y = -x + 8$ is better.

8. The table shows the mean outside temperature for each of six months and the amount of heating oil used by a family for each of those months.

Mean Outside Temperature (°F)	30	28	44	56	62	76
Heating Oil Used (gal)	112	115	94	60	35	12

a. Find an equation for a line of best fit. $y \approx -2.23x + 181$

b. Interpret the meaning of the slope and y-intercept.

c. Use your equation to predict the amount of heating oil used in a month in which the mean outside temperature is 20 °F. ≈136 gal

8b. Slope: the family will use 2.23 fewer gal/mo for each 1 °F increase in mean temp.; y-int.: the family will use 181 gal in a month when the mean temp. is 0 °F.

9. The table shows the number of customers at a coffee shop and the number of cookies sold for several days. Find an equation for a line of best fit. How well does the line represent the data? $y \approx 0.2x + 2$; very well ($r \approx 0.94$)

Customers	10	12	25	27	40	55	67	109
Cookies Sold	2	6	5	9	10	11	20	22

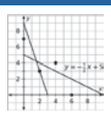
10. mod. strong pos. corr.; unlikely cause-and-effect (time spent on one activity in wk 1 probably does not affect time spent on the other activity in wk 2).

10. Some students were surveyed about how much time they spent watching television one week and how much time they spent playing video games the next week. The equation of the least-squares line for the data is $y \approx 0.76x + 1.63$ and $r \approx 0.77$. Discuss correlation and causation for the data set.

Week 1: Hours Watching Television	4	2	0	1	3	1	8	10
Week 2: Hours Playing Video Games	1	3	3	6	2	1	9	10

PRACTICE A
PRACTICE C
PRACTICE B

1. The data in the table are graphed at right along with two lines of fit.



x	0	2	4	6
y	7	3	4	0

a. Find the sum of the squares of the residuals for $y = -3x + 9$. 134
b. Find the sum of the squares of the residuals for $y = -\frac{1}{2}x + 5$. 10
c. Which line is a better fit for the data? $y = -\frac{1}{2}x + 5$

2. Use the data in the table to answer the questions that follow.

x	5	6	6.5	7.5	9
y	0	−1	3	−2	4

a. Find an equation for a line of best fit. $y \approx 0.78x - 4.54$
b. What is the correlation coefficient? 0.46
c. How well does the line represent the data? moderately well
d. Describe the correlation. weak pos. correlation

3. Use the data in the table to answer the questions that follow.

x	10	8	6	4	2
y	1	1.1	1.2	1.3	1.5

a. Find an equation for a line of best fit. $y \approx -0.06x + 1.58$
b. What is the correlation coefficient? −0.99
c. How well does the line represent the data? very well
d. Describe the correlation. strong neg. correlation

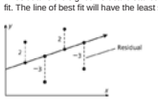
4. The table shows the number of pickles four students ate during the week versus their grades on a test. The equation of the least-squares line is $y \approx 2.11x + 79.28$, and $r \approx 0.97$. Discuss correlation and causation for the data set.

Pickles Eaten	0	2	5	10
Test Score	77	85	92	99

strong pos. correlation; unlikely cause-and-effect relationship

READING STRATEGIES

A residual is the signed vertical distance between a data point and a line of fit. The line of best fit will have the least sum of squares of residuals.



Find the line of best fit for the data set (1,1), (2, 3), (3, 2) (4, 4).

1. Enter the data into a graphing calculator.

Use **LinReg(*ax* + *b*)** to find the equation for the line of best fit.
$y = 0.8x + 0.5$

2. What is the correlation coefficient?
$r = 0.8$

3. Use the equation of best fit to complete the table below.

x	1	2	3	4
y	1.3	2.1	2.9	3.7

4. Find each residual. 0.3 −0.9 0.9 −0.3

5. Find the square of each residual. 0.09 0.81 0.81 0.09

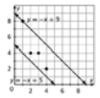
6. What is the sum of the squares? 1.8

RETEACH

The data in the table are graphed along with two lines of fit. The line that is a better fit will have the lesser *sum of the square of the residuals*. A residual is the vertical distance between a data point and a line of fit.

x	1	2	3	4
y	8	5	5	2

Find the sum of the squares of the residuals for $y = -x + 5$.



Step 1 Use the equation to find the y-values when $x = 1, 2, 3$, and 4.

x	$-x + 5 = y$
1	$-1 + 5 = 4$
2	$-2 + 5 = 3$
3	$-3 + 5 = 2$
4	$-4 + 5 = 1$

Step 2 Subtract each y-value in Step 1 from the corresponding y-value in the original table to find the residuals.
$8 - 4 = 4$ $5 - 3 = 2$ $5 - 2 = 3$ $2 - 1 = 1$

Step 3 Add the squares of the residuals.
$4^2 + 2^2 + 3^2 + 1^2 = 16 + 4 + 9 + 1 = 30$
The sum of the squares of the residuals for $y = -x + 5$ is 30.

1. Find the sum of squares of the residuals for $y = -x + 9$.
a. Use the equation to find the y-values when $x = 1, 2, 3$, and 4.

x	1	2	3	4
y	8	7	6	5

b. Subtract each y-value in Step 1 from the corresponding y-value in the original table to find the residuals.
$8 - \underline{8} = \underline{0}$ $5 - \underline{7} = \underline{-2}$ $5 - \underline{6} = \underline{-1}$ $2 - \underline{5} = \underline{-3}$
c. Add the squares of the residuals.
$\underline{0}^2 + \underline{-2}^2 + \underline{-1}^2 + \underline{-3}^2 = \underline{0} + \underline{4} + \underline{1} + \underline{9} = \underline{14}$
The line for which the sum of the squares of the residuals is lesser is the better fit for the data.
2. $y = -x + 9$ is the better fit.

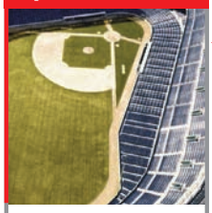
11. Write About It Tell which correlation coefficient, $r = 0.65$ or $r = -0.78$, indicates a stronger linear relationship between two variables. Explain your answer.

−0.78; −0.78 is closer to −1 than 0.65 is to 1.

12. Critical Thinking What can you conclude if the sum of the squared residuals is 0? Explain why the same conclusion might not apply when the sum of the residuals is 0. **Every data pt. lies on the least-squares line; residuals can be pos. or neg., so their sum could be 0 even when data pts. are not on the least-squares line.**

LINK

Sports



The New York Yankees opened a new stadium in 2009. Although the new stadium seats fewer fans than the old stadium (50,287 versus 56,886), the seats in the new stadium are wider, and there is more legroom between rows.

13. Sports The table shows hits and runs scored by eight New York Yankees in the 2009 baseball season. **a. $y \approx 0.48x + 12.03$**

Player	Hits	Runs
Jorge Posada	109	55
Mark Teixeira	178	103
Robinson Cano	204	103
Derek Jeter	212	107
Johnny Damon	155	107
Melky Cabrera	133	66
Nick Swisher	124	84
Hideki Matsui	125	62

a. Find the equation of the least-squares line.

b. Interpret the meaning of the slope.

c. Interpret the meaning of the *y*-intercept mathematically.

d. Describe any possible correlation for the data set. Use the correlation coefficient to support your answer.

e. Use the equation of the least-squares line from part **a** to predict how many runs a player will score if he gets 100 hits. **≈60 runs**

14. Community The table shows data about temperature and how much bottled water was sold at an annual summer festival in past years. The high temperature for the day of this year's festival is predicted to be 89 °F. The festival organizer must order bottled water in cases of 100. Find the equation of the least-squares line. Use the equation to decide how many cases the organizer should order.

$y \approx 30.43x - 1875$; 9 cases

Midtown Summer Fest						
Year	1	2	3	4	5	6
Daily High Temperature (°F)	75	82	95	92	80	84
Bottled Waters Sold	465	517	1052	940	611	625

Use the table for Exercises 15 and 16.

Regional Historical Museum						
Year	0	2	4	6	8	10
Visitors	980	1,251	1,667	1,785	2,110	2,056
Gift Shop Sales ($)	8,890	12,365	15,100	18,060	20,650	22,600

15b. Slope: each yr there will be 115.36 more visitors than the prev. yr; *y*-int.: there were 1065 visitors in yr 0.

15. Complete parts **a–d** for the relationship between the year and the number of visitors.

a. Find the equation of the least-squares line and the correlation coefficient.

b. Interpret the meaning of the slope and the *y*-intercept.

c. Is it reasonable to use your equation to make predictions? Explain.

d. Is it reasonable to say there is a cause-and-effect relationship? Explain.

No; the passage of time likely does not cause changes in the number of visitors.

16. Complete parts **a–d** above for the relationship between the number of visitors and the gift shop sales.

©Sandra Baker/Getty Images

COMMON ERROR ALERT

In **Exercise 11**, students may think that a positive *r*-value indicates a stronger correlation than a negative *r*-value. Explain that the strength of a correlation is determined by the absolute value of *r*. For example, *r*-values of –0.8 and 0.8 indicate equal strength.

Teaching Tip

Diversity In **Exercise 13**, some students may not know the official baseball definitions of *hit* and *run*. A *hit* is when a batter reaches a base safely (not including errors, walks, etc.). A *run* is scored when a player advances around all of the bases and returns safely to home plate.

Answers

13b. A player will score 0.48 run for every hit.

c. A player with no hits will score 12.03 runs.

d. There is a strong correlation between the number of hits and the number of runs, since the correlation coefficient $r \approx 0.84$

15a. $y \approx 115.36x + 1065$; $r \approx 0.96$

c. yes; $r \approx 0.96$, which is very close to 1.

16a. $y \approx 11.28x - 2239$; $r \approx 0.98$

b. slope: there is $11.28 in sales for each visitor; *y*-int.: there is −$2239 in sales if there are no visitors. (This could not actually happen.)

c. yes; $r \approx 0.98$, which is very close to 1. However, predictions for small numbers of visitors might not be useful because of the neg. *y*-int.

d. yes; more visitors likely causes more money to be spent in the gift shop.

PROBLEM SOLVING

1. The table shows the number of hours different players practice basketball each week and the number of baskets each player scored during a game.

Player	Alan	Brenda	Caleb	Shawna	Fernando	Gabriela
Hours Practiced	5	10	7	2	0	21
Baskets Scored	6	11	8	4	2	19

a. Find an equation for a line of best fit. Round decimals to the nearest tenth. $y = 0.8x + 2.3$

b. Interpret the meaning of the slope and *y*-intercept.
Slope: for each hour practiced, a player will score 0.8 baskets; *y*-int.: a player who practices 0 h will score 2.3 baskets.

c. Find the correlation coefficient. 0.998

Select the best answer.

2. Use your equation above to predict the number of baskets scored by a player who practices 40 hours a week. Round to the nearest whole number.
A 32 baskets **C**
B 33 baskets
C 34 baskets
D 35 baskets

3. Which is the best description of the correlation?
F strong positive **F**
G weak positive
H weak negative
J strong negative

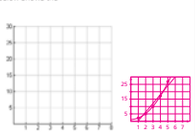
4. Given the data, what advice can you give to a player who wants to increase the number of baskets he or she scores during a game?
A Practice more hours per week. **A**
B Practice fewer hours per week.
C Practice the same hours per week.
D There is no way to increase baskets.

5. Do the data support causation, correlation, or chance?
F correlation **G**
G causation
H chance
J chance and correlation

CHALLENGE

Ellen holds fundraisers to help plant trees. The table below shows the earnings from the last five fundraisers.

Fundraiser	Earnings ($1000)
1	2
2	5
3	10
4	17
5	26



1. Plot the points on the grid at right.

2. Use a graphing calculator to find the equation of the line of best fit for the fundraiser earnings.
$y =$ 6x – 6

3. Sketch the line of best fit on your graph.

4. What is the value of *r* for this line? 0.98

5. Use the QuadReg function of a graphing calculator to find a quadratic equation to represent the fundraiser earnings.
$y = x^2 + 1$

6. Sketch this parabola on your graph.

7. What is the value of *r* for the parabola? 1

8. Explain the significance of *r* for each case. Predict how much money the next fundraiser will earn using both equations.
Possible answer: $r = 0.98$ for $y = 6x - 6$, which means there is a strong positive correlation and that the linear equation is a very good model of the data; however, $r = 1$ for $y = x^2 + 1$, which means the quadratic equation perfectly models the data. For the next fundraiser, Ellen will raise $30,000 according to the linear model $y = 6x - 6$, or $37,000 according to the quadratic model $y = x^2 + 1$.

TEST PREP DOCTOR For **Exercise 17**, remind students that negative correlation coefficients correspond to negative slopes and positive correlation coefficients correspond to positive slopes. Because the slope of a line of fit for this graph is negative, choices **C** and **D** can be eliminated.

Journal

Have students explain the meanings of *line of best fit* and *correlation coefficient*.

ALTERNATIVE ASSESSMENT

Have students plot five ordered pairs that do not lie on a line. Then ask students to visually estimate a trend line and find the squares of the residuals. Finally, ask each student to find an equation for a line of best fit, and to use the correlation coefficient to explain how well the line represents the data.

Power Presentations with PowerPoint®

4-8 Lesson Quiz

The table shows time spent on homework and number of incorrect quiz answers for several students.

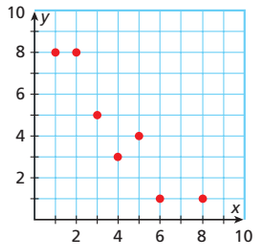
Time Spent on Homework (h)	2	4	8	10
Incorrect Answers	8	9	1	4

1. Two lines of fit are $y = -x + 11$ and $y = -0.5x + 8$. Find the sum of the squares of the residuals for each line. Which line is a better fit? 18; 20; $y = -x + 11$
2. Find an equation for a line of best fit. Interpret the meaning of the slope and y-intercept. Use your equation to predict the number of incorrect answers for 5 hours of study. $y \approx -0.8x + 10.3$; slope: for every hour of study, the number of incorrect answers decreases by 0.8; y-int.: a student who studies for 0 h will get 10.3 incorrect answers; 6.3
3. How well does the line of best fit represent the data? Explain. fairly well ($r \approx -0.79$)

Also available online

TEST PREP

17. Which could be the correlation coefficient of this graph?



(A) −1.00
(B) −0.93
(C) 0.93
(D) 1.00

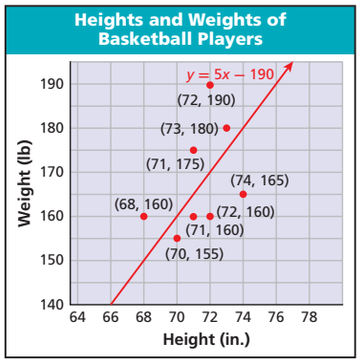
18. The table shows how much time five students studied for a test and their test scores. The equation of a line of fit for the data is $y = 5x + 60$. What is the sum of the squares of the residuals for the line of fit?

Hours Studying	0	2	4	6	8
Test Score	60	70	90	80	100

(F) 0 (G) 20 (H) 40 (J) 200

CHALLENGE AND EXTEND

19. The heights and weights of eight basketball players are graphed along with a line of fit.



a. Find the sum of the squares of the residuals. 1000

b. Find the *mean absolute deviation*. (The mean absolute deviation is the mean of the absolute values of the residuals.) Explain why the mean absolute deviation might be more useful than the sum of the squares of the residuals in some cases.

19b. 10; possible answer: The mean abs. dev. tells you the avg. vert. dist. of a data point from the line of fit.

20. Use these facts to complete the data table:
The equation of a line of fit is $y = 2x - 3$.
The sum of the residuals is 0.
The sum of the squares of the residuals is 14.
3; 2 or 0; 5

x	1	3	4	5
y	1	■	■	8

COMMON CORE

4-9 Slopes of Parallel and Perpendicular Lines

CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Also **CC.9-12.F.IF.7***

Objectives
Identify and graph parallel and perpendicular lines.

Write equations to describe lines parallel or perpendicular to a given line.

Vocabulary
parallel lines
perpendicular lines

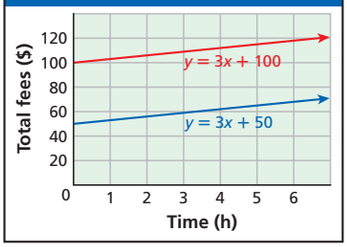
Why learn this?
Parallel lines and their equations can be used to model costs, such as the cost of a booth at a farmers' market.

To sell at a particular farmers' market for a year, there is a $100 membership fee. Then you pay $3 for each hour that you sell at the market. However, if you were a member the previous year, the membership fee is reduced to $50.

- The red line shows the total cost if you are a new member.
- The blue line shows the total cost if you are a returning member.

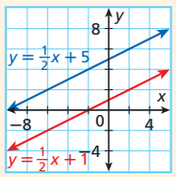
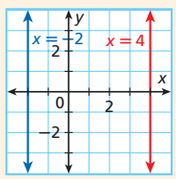
These two lines are *parallel.* **Parallel lines** are lines in the same plane that have no points in common. In other words, they do not intersect.

Farmers' Market Fees



Know it! Note

Parallel Lines

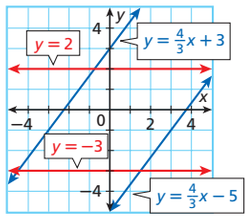
WORDS	Two different nonvertical lines are parallel if and only if they have the same slope.	All different vertical lines are parallel.
GRAPH		

EXAMPLE 1 **Identifying Parallel Lines**

Identify which lines are parallel.

A $y = \frac{4}{3}x + 3; y = 2; y = \frac{4}{3}x - 5; y = -3$

The lines described by $y = \frac{4}{3}x + 3$ and $y = \frac{4}{3}x - 5$ both have slope $\frac{4}{3}$. These lines are parallel.The lines described by $y = 2$ and $y = -3$ both have slope 0. These lines are parallel.



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1 Introduce

EXPLORATION

Slopes of Parallel and Perpendicular Lines

You will need graph paper or a graphing calculator for this Exploration.

1. Graph $y = 2x + 3$ and $y = 2x - 2$ so that you can see both graphs at the same time. What do you notice about the lines? The lines are parallel.
2. Compare the slopes of the lines. They are the same.
3. Repeat this process with $y = \frac{1}{3}x + 2$ and $y = \frac{1}{3}x$. What do you notice about the lines? What are the slopes of the lines? The lines are parallel; each line has slope $\frac{1}{3}$.

THINK AND DISCUSS

4. **Discuss** any conjectures that you can make based on your findings. Two (nonvertical) lines do not intersect (are parallel) if and only if they have the same slope.
5. **Describe** what you would expect to be true, based on your conjectures, about the graphs of $y = \frac{2}{3}x - 2$ and $y = \frac{2}{3}x - 5$. The lines will be parallel.

Motivate

Have students use a graphing calculator to graph $y = 4x + 3$ and $y = 4x - 1$. Ask students where the lines intersect. They do not intersect.

Have students graph $y = 4x + 3$ and $y = -\frac{1}{4}x + 3$.

Tell students to estimate the measures of the angles at the intersection of the two lines. 90°

Explorations and answers are provided in your online resources.

4-9 Organizer

Pacing: Traditional $1\frac{1}{2}$ days
Block $\frac{3}{4}$ day

Objectives: Identify and graph parallel and perpendicular lines.

Write equations to describe lines parallel or perpendicular to a given line.

Algebra Lab
In *Algebra Lab Activities*

Online Edition
Tutorial Videos, Interactivity

Countdown

Power Presentations with PowerPoint®

Warm Up

Find the reciprocal.

1. 2 $\frac{1}{2}$
2. $\frac{1}{3}$ 3
3. $-\frac{3}{4}$ $-\frac{4}{3}$

Find the slope of the line that passes through each pair of points.

4. (2, 2) and (−1, 3) $-\frac{1}{3}$
5. (3, 4) and (4, 6) 2
6. (5, 1) and (0, 0) $\frac{1}{5}$

Also available online

Math Humor

Q: What did the lines do at the gym to avoid each other?

A: They used the parallel bars.

COMMON CORE

CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems…
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph…
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Learn It Online
State Resources Online

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Additional Examples

Example 1

Identify which lines are parallel.

A. $y = \frac{5}{3}x - 2$; $y = x$;
$y = \frac{5}{3}x + 4$; $y = x + 1$

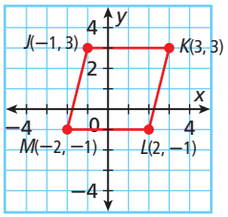
$y = \frac{5}{3}x - 2$ and $y = \frac{5}{3} + 4$; $y = x$ and $y = x + 1$

B. $y = 2x - 3$; $y = -\frac{2}{3}x + 3$; $2x + 3y = 8$; $y + 1 = 3(x - 3)$

$y = -\frac{2}{3}x + 3$ and $2x + 3y = 8$

Example 2

Show that *JKLM* is a parallelogram.



Slope of $\overline{JK} = 0$; slope of $\overline{KL} = 4$; slope of $\overline{ML} = 0$; slope of $\overline{JM} = 4$; $\overline{JK}$ is par. to $\overline{ML}$ because they are both horiz. $\overline{KL}$ is par. to $\overline{JM}$ because they have the same slope. Since opposite sides are par., *JKLM* is a parallelogram.

Also available online

Identify which lines are parallel.

B $y = 3x + 2$; $y = -\frac{1}{2}x + 4$; $x + 2y = -4$; $y - 5 = 3(x - 1)$

Write all equations in slope-intercept form to determine the slopes.

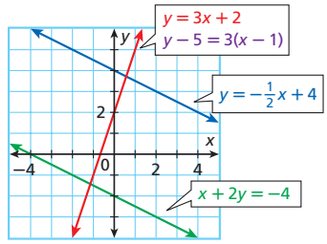
$y = 3x + 2$	$y = -\frac{1}{2}x + 4$
slope-intercept form ✓	slope-intercept form ✓

$x + 2y = -4$
$-x \quad -x$
$2y = -x - 4$
$\frac{2y}{2} = \frac{-x - 4}{2}$
$y = -\frac{1}{2}x - 2$

$y - 5 = 3(x - 1)$
$y - 5 = 3x - 3$
$+5 \quad +5$
$y = 3x + 2$

The lines described by $y = 3x + 2$ and $y - 5 = 3(x - 1)$ have the same slope, but they are not parallel lines. They are the same line.

The lines described by $y = -\frac{1}{2}x + 4$ and $x + 2y = -4$ represent parallel lines. They each have slope $-\frac{1}{2}$.



CHECK IT OUT!

Identify which lines are parallel.

1a. $y = 2x + 2$; $y = 2x + 1$; $y = -4$; $x = 1$

1b. $y = \frac{3}{4}x + 8$; $-3x + 4y = 32$; $y = 3x$; $y - 1 = 3(x + 2)$

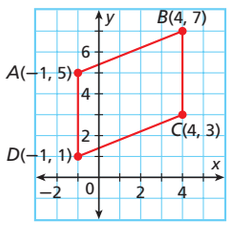
1a. $y = 2x + 2$ and $y = 2x + 1$ **1b. $y = 3x$ and $y - 1 = 3(x + 2)$**

EXAMPLE 2 ***Geometry Application***

Geometry

Show that *ABCD* is a parallelogram.

Use the ordered pairs and the slope formula to find the slopes of $\overline{AB}$ and $\overline{CD}$.



slope of $\overline{AB} = \frac{7 - 5}{4 - (-1)} = \frac{2}{5}$

slope of $\overline{CD} = \frac{3 - 1}{4 - (-1)} = \frac{2}{5}$

$\overline{AB}$ is parallel to $\overline{CD}$ because they have the same slope.

$\overline{AD}$ is parallel to $\overline{BC}$ because they are both vertical.

Therefore, *ABCD* is a parallelogram because both pairs of opposite sides are parallel.

Remember!

In a parallelogram, opposite sides are parallel.

2. slope of $\overline{AB} = 0$; slope of $\overline{BC} = \frac{5}{3}$; slope of $\overline{CD} = 0$; slope of $\overline{AD} = \frac{5}{3}$; $\overline{AB}$ is par. to $\overline{CD}$ because they have the same slope. $\overline{AD}$ is par. to $\overline{BC}$ because they have the same slope. Since opp. sides are par., *ABCD* is a parallelogram.

CHECK IT OUT!

2. Show that the points $A(0, 2)$, $B(4, 2)$, $C(1, -3)$, and $D(-3, -3)$ are the vertices of a parallelogram.

Questioning Strategies

EXAMPLE 1

- What is always true about the slopes of parallel lines?
- Can functions whose graphs are parallel lines have the same *y*-intercept? Explain.

EXAMPLE 2

- Why is it not necessary to calculate the slopes of horizontal and vertical lines?

2 Teach

Guided Instruction

Start the lesson by reviewing how to rewrite equations that are in standard and point-slope form in slope-intercept form. Then work through **Examples 1** and **2.** Prior to **Example 3,** remind students that reciprocals, like $\frac{3}{7}$ and $\frac{7}{3}$, have a product of 1. Opposite reciprocals, like $\frac{2}{3}$ and $-\frac{3}{2}$, have a product of -1. Then work through the remaining examples.

Reaching All Learners

Through Communication

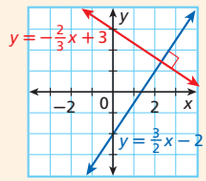
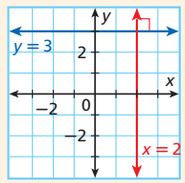
Divide the class into small groups. Have the groups develop and present one of the following:

- a 60-second TV commercial about parallel or perpendicular lines
- a two-minute news segment about how to use parallel and perpendicular lines in geometry
- a brief infomercial about the equations of parallel and perpendicular lines

Perpendicular lines are lines that intersect to form right angles (90°).

Know it! Note

Perpendicular Lines

WORDS	Two nonvertical lines are perpendicular if and only if the product of their slopes is −1.	Vertical lines are perpendicular to horizontal lines.
GRAPH		

EXAMPLE 3 Identifying Perpendicular Lines

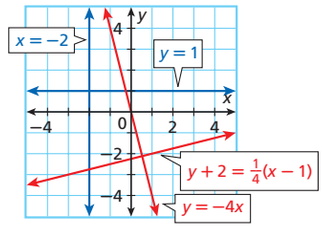
Identify which lines are perpendicular: $x = -2$; $y = 1$; $y = -4x$; $y + 2 = \frac{1}{4}(x + 1)$.

The graph described by $x = -2$ is a vertical line, and the graph described by $y = 1$ is a horizontal line. These lines are perpendicular.

The slope of the line described by $y = -4x$ is -4. The slope of the line described by $y + 2 = \frac{1}{4}(x - 1)$ is $\frac{1}{4}$.

$(-4)\left(\frac{1}{4}\right) = -1$

These lines are perpendicular because the product of their slopes is -1.



4. slope of $\overline{PQ} = 2$; slope of $\overline{QR} = -1$; slope of $\overline{PR} = -\frac{1}{2}$; $\overline{PQ}$ is perp. to $\overline{PR}$ because the product of their slopes is -1. Since PQR contains a rt. angle, PQR is a rt. triangle.

CHECK IT OUT!

3. Identify which lines are perpendicular: $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$.

$y = -4$ and $x = 3$; $y - 6 = 5(x + 4)$ and $y = -\frac{1}{5}x + 2$

EXAMPLE 4 Geometry Application

Geometry

Show that PQR is a right triangle.

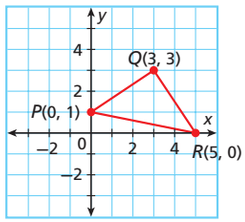
If PQR is a right triangle, $\overline{PQ}$ will be perpendicular to $\overline{QR}$.

slope of $\overline{PQ} = \frac{3 - 1}{3 - 0} = \frac{2}{3}$

slope of $\overline{QR} = \frac{3 - 0}{3 - 5} = \frac{3}{-2} = -\frac{3}{2}$

$\overline{PQ}$ is perpendicular to $\overline{QR}$ because $\frac{2}{3}\left(-\frac{3}{2}\right) = -1$.

Therefore, PQR is a right triangle because it contains a right angle.



Helpful Hint

A right triangle contains one right angle. In Example 4, $\angle P$ and $\angle R$ are clearly not right angles, so the only possibility is $\angle Q$.

CHECK IT OUT!

4. Show that $P(1, 4)$, $Q(2, 6)$, and $R(7, 1)$ are the vertices of a right triangle.

Teaching Tip

Geometry Have students think of geometric shapes that have parallel or perpendicular sides. Their list could include squares, rhombuses, rectangles, and trapezoids, as well as parallelograms and right triangles.

COMMON ERROR ALERT

Students might always use the coefficient of x for slope, regardless of the form of the equation. Tell students to always check the form of the equation first, because the coefficient of x may change when the equation is written in slope-intercept form.

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Additional Examples

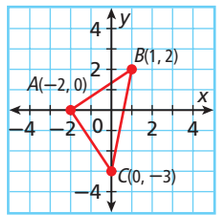
Example 3

Identify which lines are perpendicular:

$y = 3$; $x = -2$; $y = 3x$; $y = -\frac{1}{3}(x - 4)$. $y = 3$ and $x = -2$; $y = 3x$ and $y = -\frac{1}{3}(x - 4)$

Example 4

Show that ABC is a right triangle.



Slope of $\overline{AB} = \frac{2}{3}$; slope of $\overline{AC} = -\frac{3}{2}$; $\overline{AB}$ is perp. to $\overline{AC}$ because the product of their slopes is -1. Since ABC contains a rt. angle, ABC is a rt. triangle.

Also available online

MATHEMATICAL PRACTICES

Questioning Strategies

EXAMPLE 3

- What is true about the slopes of perpendicular lines?
- Can perpendicular lines have the same y-intercept? Explain.

EXAMPLE 4

- What is true about the intersection of two perpendicular lines?
- What is the maximum number of right angles a triangle could contain?

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Additional Examples

Example 5

A. Write an equation in slope-intercept form for the line that passes through (4, 10) and is parallel to the line described by $y = 3x + 8$. $y = 3x - 2$

B. Write an equation in slope-intercept form for the line that passes through (2, −1) and is perpendicular to the line described by $y = 2x - 5$. $y = -\frac{1}{2}x$

Also available online

Questioning Strategies

EXAMPLE 5

- How do you know what to substitute for *m* if you are writing an equation for a line parallel to a given line? for a line perpendicular to a given line?

Teaching Tip **Visual** Encourage students to check their work in **Example 5** by graphing.

Teaching Tip **Technology** When using a graphing calculator, lines may not appear perpendicular in the standard window. This is because units on the *x*-axis are larger than units on the *y*-axis. To view a more accurate graph, have students use the square window by pressing ZOOM and selecting 5:ZSquare.

EXAMPLE 5 Writing Equations of Parallel and Perpendicular Lines

A **Write an equation in slope-intercept form for the line that passes through (4, 5) and is parallel to the line described by $y = 5x + 10$.**

Step 1 Find the slope of the line.

$y = 5x + 10$ *The slope is 5.*

The parallel line also has a slope of 5.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

$y - 5 = 5(x - 4)$ *Substitute 5 for m, 4 for x_1, and 5 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 5 = 5(x - 4)$

$y - 5 = 5x - 20$ *Distributive Property*

$y = 5x - 15$ *Addition Property of Equality*

B **Write an equation in slope-intercept form for the line that passes through (3, 2) and is perpendicular to the line described by $y = 3x - 1$.**

Step 1 Find the slope of the line.

$y = 3x - 1$ *The slope is 3.*

The perpendicular line has a slope of $-\frac{1}{3}$, because $3\left(-\frac{1}{3}\right) = -1$.

Step 2 Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$ *Use point-slope form.*

$y - 2 = -\frac{1}{3}(x - 3)$ *Substitute $-\frac{1}{3}$ for m, 3 for x_1, and 2 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 2 = -\frac{1}{3}(x - 3)$

$y - 2 = -\frac{1}{3}x + 1$ *Distributive Property*

$y = -\frac{1}{3}x + 3$ *Addition Property of Equality*

Helpful Hint

If you know the slope of a line, the slope of a perpendicular line will be the "opposite reciprocal."

$\frac{2}{3} \rightarrow -\frac{3}{2}$

$\frac{1}{5} \rightarrow -5$

$-7 \rightarrow \frac{1}{7}$

CHECK IT OUT!

5a. Write an equation in slope-intercept form for the line that passes through (5, 7) and is parallel to the line described by $y = \frac{4}{5}x - 6$.

5b. Write an equation in slope-intercept form for the line that passes through (−5, 3) and is perpendicular to the line described by $y = 5x$.

5a. $y = \frac{4}{5}x + 3$ **5b.** $y = -\frac{1}{5}x + 2$

THINK AND DISCUSS

1. Are the lines described by $y = \frac{1}{2}x$ and $y = 2x$ perpendicular? Explain.

2. Describe the slopes and *y*-intercepts when two nonvertical lines are parallel.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example and describe the slopes.

Parallel lines — Perpendicular lines

Know it! Note

3 Close

Summarize

Remind students that they will need to know the slopes of lines to determine whether the lines are parallel, perpendicular, or neither. Tell students that equations can be presented in many ways, and that they may need to rewrite equations in slope-intercept form before deciding on the value of the slope.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. No; the product of their slopes is 1, not −1.
2. The slopes are the same, and the *y*-intercepts are different.
3. See Additional Answers.

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Homework Help Online
Parent Resources Online

GUIDED PRACTICE

1. **Vocabulary** ___?___ lines have the same slope. (*Parallel* or *Perpendicular*) **Parallel**

SEE EXAMPLE 1

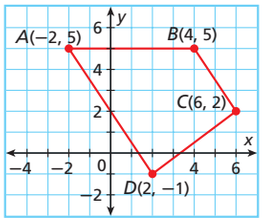
Identify which lines are parallel.

2. $y = 6$; $y = 6x + 5$; $y = 6x - 7$; $y = -8$ **$y = 6x + 5$ and $y = 6x - 7$; $y = 6$ and $y = -8$**

3. $y = \frac{3}{4}x - 1$; $y = -2x$; $y - 3 = \frac{3}{4}(x - 5)$; $y - 4 = -2(x + 2)$

SEE EXAMPLE 2

4. **Geometry** Show that *ABCD* is a trapezoid. (*Hint:* In a trapezoid, exactly one pair of opposite sides is parallel.)



SEE EXAMPLE 3

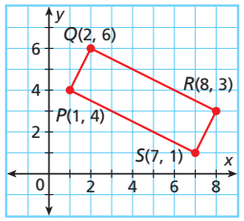
Identify which lines are perpendicular.

5. $y = \frac{2}{3}x - 4$; $y = -\frac{3}{2}x + 2$; $y = -1$; $x = 3$

6. $y = -\frac{3}{7}x - 4$; $y - 4 = -7(x + 2)$; $y - 1 = \frac{1}{7}(x - 4)$; $y - 7 = \frac{7}{3}(x - 3)$

SEE EXAMPLE 4

7. **Geometry** Show that *PQRS* is a rectangle. (*Hint:* In a rectangle, all four angles are right angles.)



SEE EXAMPLE 5

8. Write an equation in slope-intercept form for the line that passes through (5, 0) and is perpendicular to the line described by $y = -\frac{5}{2}x + 6$. **$y = \frac{2}{5}x - 2$**

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12	2
13–15	3
16	4
17	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

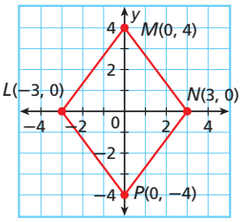
Identify which lines are parallel.

9. $x = 7$; $y = -\frac{5}{6}x + 8$; $y = -\frac{5}{6}x - 4$; $x = -9$

10. $y = -x$; $y - 3 = -1(x + 9)$; $y - 6 = \frac{1}{2}(x - 14)$; $y + 1 = \frac{1}{2}x$

11. $y = -3x + 2$; $y = \frac{1}{2}x - 1$; $-x + 2y = 17$; $3x + y = 27$

12. **Geometry** Show that *LMNP* is a parallelogram.



Identify which lines are perpendicular.

13. $y = 6x$; $y = \frac{1}{6}x$; $y = -\frac{1}{6}x$; $y = -6x$

14. $y - 9 = 3(x + 1)$; $y = -\frac{1}{3}x + 5$; $y = 0$; $x = 6$

15. $x - 6y = 15$; $y = 3x - 2$; $y = -3x - 3$; $y = -6x - 8$; $3y = -x - 11$
$x - 6y = 15$ and $y = -6x - 8$; $y = 3x - 2$ and $3y = -x - 11$

Answers

7. slope of $\overline{PQ} = 2$; slope of $\overline{QR} = -\frac{1}{2}$; slope of $\overline{RS} = 2$; slope of $\overline{PS} = -\frac{1}{2}$
$\overline{PQ}$ is perp. to $\overline{QR}$ because the product of their slopes is -1. $\overline{QR}$ is perp. to $\overline{RS}$ because the product of their slopes is -1. $\overline{RS}$ is perp. to $\overline{PS}$ because the product of their slopes is -1. $\overline{PS}$ is perp. to $\overline{PQ}$ because the product of their slopes is -1. Therefore all the angles are rt. angles, and *PQRS* is a rectangle.
9. $x = 7$ and $x = -9$; $y = -\frac{5}{6}x + 8$ and $y = -\frac{5}{6}x - 4$
10. $y = -x$ and $y - 3 = -1(x + 9)$
11. $y = -3x + 2$ and $3x + y = 27$; $y = \frac{1}{2}x - 1$ and $-x + 2y = 17$
12. slope of $\overline{LM} = \frac{4}{3}$; slope of $\overline{MN} = -\frac{4}{3}$; slope of $\overline{PN} = \frac{4}{3}$; slope of $\overline{LP} = -\frac{4}{3}$
$\overline{LM}$ is par. to $\overline{PN}$ because they have the same slope. $\overline{MN}$ is par. to $\overline{LP}$ because they have the same slope. Since opp. sides are par., *LMNP* is a parallelogram.
13. $y = 6x$ and $y = -\frac{1}{6}x$; $y = \frac{1}{6}x$ and $y = -6x$
14. $y - 9 = 3(x + 1)$ and $y = -\frac{1}{3}x + 5$; $y = 0$ and $x = 6$

Assignment Guide

Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
Basic 9–15, 18–21
Average 9–15, 18–21, 49
Advanced 9–15, 18–21, 49, 56, 57

If you finished Examples **1–5**
Basic 9–45, 51–54
Average 9–17, 18–44 even, 46–54, 56–58
Advanced 9–17, 18–44 even, 46–58

Homework Quick Check
Quickly check key concepts.
Exercises: 10, 12, 14, 16, 26, 42

Answers

3. $y = \frac{3}{4}x - 1$ and $y - 3 = \frac{3}{4}(x - 5)$
4. slope of $\overline{AD} = -\frac{3}{2}$; slope of $\overline{BC} = -\frac{3}{2}$; $\overline{AD}$ and $\overline{BC}$ are par. because they have the same slope. Therefore, *ABCD* is a trapezoid.
5. $y = \frac{2}{3}x - 4$ and $y = -\frac{3}{2}x + 2$; $y = -1$ and $x = 3$
6. $y = -\frac{3}{7}x - 4$ and $y - 7 = \frac{7}{3}(x - 3)$; $y - 4 = -7(x + 2)$ and $y - 1 = \frac{1}{7}(x - 4)$

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 51

Construct viable arguments and critique the reasoning of others. Exercises 4, 7, 12, 16, 48, 50, 55, 58

Attend to precision. Exercise 49

Look for and make use of structure. Exercises 18–21

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MULTI-STEP TEST PREP **Exercise 51** involves writing and graphing functions. This exercise prepares students for the Multi-Step Test Prep.

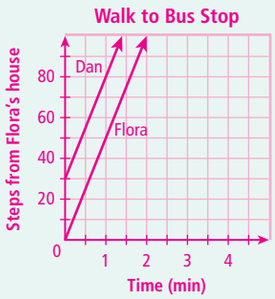
Answers

16. slope of $\overline{AB} = -\frac{1}{4}$; slope of $\overline{BC} = 4$; $\overline{AB}$ is perp. to $\overline{BC}$ because the product of their slopes is -1. Since ABC contains a rt. angle, ABC is a rt. triangle.

48. Possible answer: No; par. lines have no points in common. If they had the same y-int., they would both intersect the y-axis at the same place, and they could not be par.

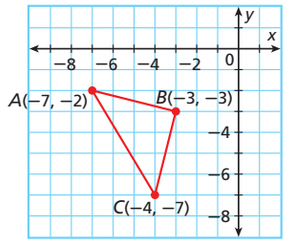
50. Possible answer: First find the slope of $y - 3 = -6(x - 3)$. Since it is written in point-slope form, you can immediately tell that the slope is -6. Then find the y-int. of $y - 3 = -6(x - 3)$ by solving for y: $y = -6x + 21$. So the y-int. is 21. Choose any other y-int. b and write the equation $y = -6x + b$. This line will be par. to $y - 3 = -6(x - 3)$.

51c. No; the lines are par. and never intersect, so Flora and Dan will never be at the same place at the same time during their walk. Because Dan is walking at the same pace as Flora, Flora will not be able to catch up.



22. $y = 3x + 4$
23. $y = \frac{1}{2}x - 5$
24. $y = \frac{1}{4}x - 1$
25. $y = 2x + 5$
26. $y = \frac{5}{2}x - \frac{25}{2}$
27. $y = 3x + 13$
28. $y = 4$
29. $y = -x + 5$
30. $y = -\frac{2}{3}x + \frac{23}{3}$
31. $y = 4x - 23$
32. $y = \frac{1}{2}x - 4$
33. $y = -\frac{3}{4}x$
34. $y = \frac{1}{3}x - 4$
35. $y = -x + 1$
36. $y = -\frac{4}{3}x - 3$
37. $y = \frac{2}{5}x - \frac{31}{5}$
38. $y = \frac{1}{3}x - \frac{14}{3}$
39. $y = -\frac{1}{5}x - \frac{11}{5}$
40. $y = \frac{3}{2}x - 1$
41. $y = -\frac{1}{2}x - \frac{1}{2}$
42. $y = 4x - 11$
43. $y = \frac{1}{2}x + 6$
44. $y = -x + 5$
45. $y = x - 3$

16. **Geometry** Show that ABC is a right triangle.



17. Write an equation in slope-intercept form for the line that passes through $(0, 0)$ and is parallel to the line described by $y = -\frac{6}{7}x + 1$. $y = -\frac{6}{7}x$

Without graphing, tell whether each pair of lines is parallel, perpendicular, or neither.

18. $x = 2$ and $y = -5$ **perp.**
19. $y = 7x$ and $y - 28 = 7(x - 4)$ **neither**
20. $y = 2x - 1$ and $y = \frac{1}{2}x + 2$ **neither**
21. $y - 3 = \frac{1}{4}(x - 3)$ and $y + 13 = \frac{1}{4}(x + 1)$ **par.**

Write an equation in slope-intercept form for the line that is parallel to the given line and that passes through the given point.

22. $y = 3x - 7$; $(0, 4)$
23. $y = \frac{1}{2}x + 5$; $(4, -3)$
24. $4y = x$; $(4, 0)$
25. $y = 2x + 3$; $(1, 7)$
26. $5x - 2y = 10$; $(3, -5)$
27. $y = 3x - 4$; $(-2, 7)$
28. $y = 7$; $(2, 4)$
29. $x + y = 1$; $(2, 3)$
30. $2x + 3y = 7$; $(4, 5)$
31. $y = 4x + 2$; $(5, -3)$
32. $y = \frac{1}{2}x - 1$; $(0, -4)$
33. $3x + 4y = 8$; $(4, -3)$

Write an equation in slope-intercept form for the line that is perpendicular to the given line and that passes through the given point.

34. $y = -3x + 4$; $(6, -2)$
35. $y = x - 6$; $(-1, 2)$
36. $3x - 4y = 8$; $(-6, 5)$
37. $5x + 2y = 10$; $(3, -5)$
38. $y = 5 - 3x$; $(2, -4)$
39. $-10x + 2y = 8$; $(4, -3)$
40. $2x + 3y = 7$; $(4, 5)$
41. $4x - 2y = -6$; $(3, -2)$
42. $-2x - 8y = 16$; $(4, 5)$
43. $y = -2x + 4$; $(-2, 5)$
44. $y = x - 5$; $(0, 5)$
45. $x + y = 2$; $(8, 5)$

46. Write an equation describing the line that is parallel to the y-axis and that is 6 units to the right of the y-axis. $x = 6$

47. Write an equation describing the line that is perpendicular to the y-axis and that is 4 units below the x-axis. $y = -4$

48. **Critical Thinking** Is it possible for two linear functions whose graphs are parallel lines to have the same y-intercept? Explain.

49. **Estimation** Estimate the slope of a line that is perpendicular to the line through $(2.07, 8.95)$ and $(-1.9, 25.07)$. Possible answer: about $\frac{1}{4}$

50. **Write About It** Explain in words how to write an equation in slope-intercept form that describes a line parallel to $y - 3 = -6(x - 3)$.

MULTI-STEP TEST PREP

51. a. Flora walks from her home to the bus stop at a rate of 50 steps per minute. Write a rule that gives her distance from home (in steps) as a function of time. $y = 50x$

b. Flora's neighbor Dan lives 30 steps closer to the bus stop. He begins walking at the same time and at the same pace as Flora. Write a rule that gives Dan's distance from *Flora's* house as a function of time. $y = 50x + 30$

c. Will Flora meet Dan along the walk? Use a graph to help explain your answer.

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PRACTICE A

PRACTICE C

PRACTICE B

Identify which lines are parallel.

1. $y = 3x + 4$; $y = 4$; $y = 3x$; $y = 3$

$y = 3x + 4$ and $y = 3x$; $y = 4$ and $y = 3$

2. $y = \frac{1}{2}x + 4$; $x = \frac{1}{2}$; $2x + y = 1$; $y = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 4$ and $y = \frac{1}{2}x + 1$

3. Find the slope of each segment.

slope of $\overline{AB}$: $-\frac{2}{3}$

slope of $\overline{AD}$: undefined

slope of $\overline{DC}$: $-\frac{2}{3}$

slope of $\overline{BC}$: undefined

Explain why $ABCD$ is a parallelogram.

The opposite sides have the same slope which means they are parallel. A quadrilateral is a parallelogram if the opposite sides are parallel.

Identify which lines are perpendicular.

4. $y = 5$; $y = \frac{1}{8}x$; $x = 2$; $y = 8x - 5$

$y = 5$ and $x = 2$

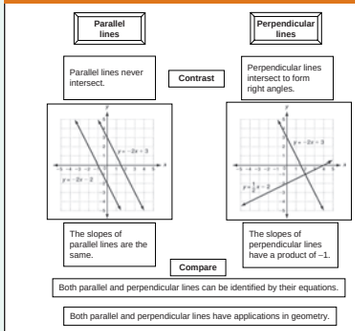
5. $y = -2$; $y = -\frac{1}{2}x - 4$; $y - 4 = 2(x + 3)$; $y = -2x$

$y = -\frac{1}{2}x - 4$ and $y - 4 = 2(x + 3)$

6. Show that ABC is a right triangle.

slope of $\overline{AB} = \frac{1}{4}$; slope of $\overline{BC} = -4$; $\overline{AB}$ is perpendicular to $\overline{BC}$ because $\frac{1}{4}(-4) = -1$. ABC is a right triangle because it contains a right angle.

READING STRATEGIES



Possible answers are given for 1 and 2.

1. Write an equation of a line parallel to $y = 3x + 4$. $y = 3x + 3$
2. Write an equation of a line perpendicular to $y = 3x + 4$. $y = -\frac{1}{3}x + 3$

$A(9, -4)$, $B(-3, 0)$ and $C(1, 4)$ are the vertices of a triangle.

3. Find the slope of $\overline{AB}$. $-\frac{1}{3}$
4. Find the slope of $\overline{BC}$. 1
5. Find the slope of $\overline{AC}$. -1
6. Is ABC a right triangle? Why? yes, $\overline{BC}$ and $\overline{AC}$ are perpendicular because $1(-1) = -1$.

RETEACH

Two lines are **parallel** if they lie in the same plane and have no points in common. The lines will never intersect.

Identify which lines are parallel.

$y = -2x + 4$; $y = 3x + 4$; $y = -2x - 1$

If lines have the same slope, but different y-intercepts, they are parallel lines.

$y = -2x + 4$;	$y = 3x + 4$;	$y = -2x - 1$
$m = -2$,	$m = 3$	$m = -2$
$b = 4$	$b = 4$	$b = -1$

$y = -2x + 4$ and $y = -2x - 1$ are parallel.

Two lines are **perpendicular** if they intersect to form right angles.

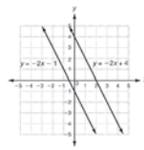
Identify which lines are perpendicular.

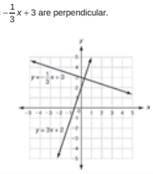
If the product of the slopes of two lines is -1, the two lines are perpendicular.

$y = -3x + 1$; $y = 3x + 2$; $y = -\frac{1}{3}x + 3$

$m = -3$	$m = 3$	$m = -\frac{1}{3}$

Because $3\left(-\frac{1}{3}\right) = -1$, $y = 3x + 2$ and $y = -\frac{1}{3}x + 3$ are perpendicular.





Identify which two lines are parallel. Then graph the parallel lines.

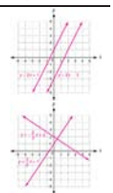
1. $y = 4x + 2$; $y = 2x + 1$; $y = 2x - 3$

$y = 2x + 1$; $y = 2x - 3$

Identify which two lines are perpendicular. Then graph the perpendicular lines.

2. $y = -\frac{2}{3}x + 2$; $y = \frac{3}{2}x + 1$; $y = \frac{2}{3}x - 3$

$y = -\frac{2}{3}x + 2$; $y = \frac{3}{2}x + 1$



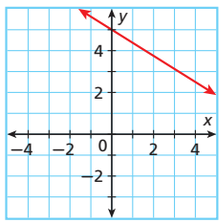
TEST PREP

52. Which describes a line parallel to the line described by $y = -3x + 2$?

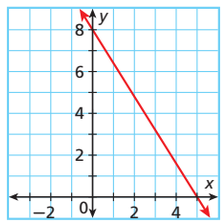
Ⓐ $y = -3x$ Ⓑ $y = \frac{1}{3}x$ Ⓒ $y = 2 - 3x$ Ⓓ $y = \frac{1}{3}x + 2$

53. Which describes a line passing through $(3, 3)$ that is perpendicular to the line described by $y = \frac{3}{5}x + 2$?

Ⓕ



Ⓗ



Ⓖ $y = \frac{5}{3}x - 2$ Ⓙ $y = \frac{3}{5}x + \frac{6}{5}$

54. Gridded Response The graph of a linear function $f(x)$ is parallel to the line described by $2x + y = 5$ and contains the point $(6, -2)$. What is the y-intercept of $f(x)$? **10**

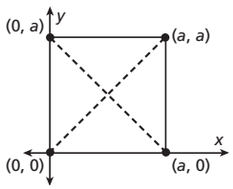
CHALLENGE AND EXTEND

55. Three or more points that lie on the same line are called *collinear points*. Explain why the points A, B, and C must be collinear if the line containing A and B has the same slope as the line containing B and C.

56. The lines described by $y = (a + 12)x + 3$ and $y = 4ax$ are parallel. What is the value of a? **4**

57. The lines described by $y = (5a + 3)x$ and $y = -\frac{1}{2}x$ are perpendicular. What is the value of a? $-\frac{1}{5}$

58. Geometry The diagram shows a square in the coordinate plane. Use the diagram to show that the diagonals of a square are perpendicular. **The slope of one diagonal is $\frac{a-0}{a-0} = \frac{a}{a} = 1$. The slope of the other diagonal is $\frac{0-a}{a-0} = \frac{-a}{a} = -1$. The product of the slopes is $1(-1) = -1$, so the diagonals are perp.**



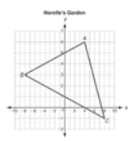
PROBLEM SOLVING

Write the correct answer.

1. Hamid is making a stained-glass window. He needs a piece of glass that is a perfect parallelogram. Hamid lays a piece of glass that he has cut on a coordinate grid. Show that the glass is in the shape of a parallelogram.

The top and bottom are parallel because they are both horizontal. The sides are parallel because they both have a slope of $-\frac{5}{3}$. It is a parallelogram because both pairs of opposite sides are parallel.

2. Norelle's garden is shown at right. Is her garden in the shape of a right triangle? Justify your answer.



The slope of AB is $\frac{1}{4}$, the slope of AC is $-\frac{7}{4}$, and the slope of BC is $-\frac{1}{4}$. None of the slopes have a product of −1 so no sides are perpendicular.

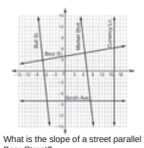
The graph shows a street map.
Use it to answer questions 3–5.

3. The district plans to add Industrial Road next year. It will run perpendicular to Smith Ave. and pass through (−14, 2). What equation will describe the location of Industrial Road?

A $y = 14 - x$ C $y = -14$
B $y = x - 14$ **D** $x = -14$

4. In two years, the business district plans to add Stock Street. It will run parallel to Market Blvd. and pass through (−1, 5). What equation will describe the location of Stock Street?

F $y = -7x + 12$ H $y = \frac{1}{7}x + \frac{34}{7}$
G $y = -7x - 2$ J $y = \frac{1}{7}x + \frac{36}{7}$



5. What is the slope of a street parallel to Bear Street?

A −7 **C** $\frac{1}{7}$
B $-\frac{1}{7}$ D 7

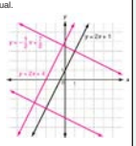
CHALLENGE

Two facts about parallel and perpendicular lines are summarized below.

If two nonvertical lines are parallel, then their slopes are equal.

If two nonvertical lines are perpendicular, then their slopes are negative reciprocals of one another.

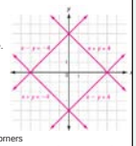
You can use these facts if you want to construct special polygons on the coordinate plane.



In Exercises 1–4, use the grid and the graph of $y = 2x + 1$.

1. a. Write an equation in slope-intercept form for the line parallel to the graph of $y = 2x + 1$ and having a y-intercept of 4. **$y = 2x + 4$**
   b. Graph your equation on the grid at right above.
2. a. Find the y-coordinate of the point on the graph of $y = 2x + 1$ for which $x = -2$. **−3**
   b. Find an equation for the line containing the point found in Part a and perpendicular to the graph of $y = 2x + 1$ **$y = -\frac{1}{2}x - 4$**
   c. Graph your equation from Part b on the grid at right above.
3. a. Find the y-coordinate of the point on the graph of $y = 2x + 1$ for which $x = 1$. **3**
   b. Find an equation for the line containing the point found in Part a and perpendicular to the graph of $y = 2x + 1$. **$y = -\frac{1}{2}x + \frac{7}{2}$**
   c. Graph your equation from Part b on the grid at right above.
4. Identify the polygon that you formed. Explain your response.

It is a rectangle because it has four right angles.



In Exercises 5 and 6, use the grid at right.

5. Suppose that you want to make a square using the graph of $x + y = 4$ to help determine one side of the square. Write equations in standard form to determine a square with corners at (4, 0), (0, 4), (−4, 0) and (0, −4). **$x - y = -4$; $x + y = -4$; $x - y = 4$; $x + y = 4$**
6. On the grid at right, graph each of the equations that you wrote in Exercise 5.

TEST PREP DOCTOR

Students who chose **C** for **Exercise 52** chose the same line, not a parallel line.

Students who chose **J** in **Exercise 53** found a line parallel to the given line, not perpendicular to it.

Answers

55. See Additional Answers.

Journal

Have students explain how to write the equation of a line that is perpendicular to a given line, such as $y - 2 = 3(x + 1)$.

ALTERNATIVE ASSESSMENT

Have students write three equations: one in standard form, one in slope-intercept form, and one in point-slope form. Have them write equations of lines parallel to each line. Then have them write equations of lines perpendicular to each line.

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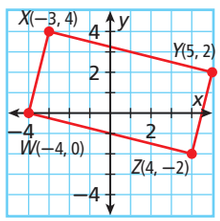
4-9 Lesson Quiz

Write an equation in slope-intercept form for the line described.

1. contains the point $(8, -12)$ and is parallel to $y = -\frac{3}{4}x - 9$ $y = -\frac{3}{4}x - 6$
2. contains the point $(4, -3)$ and is perpendicular to $y = 4x + 5$ $y = -\frac{1}{4}x - 2$
3. Show that $WXYZ$ is a rectangle.



Slope of $\overline{XY} = -\frac{1}{4}$; slope of $\overline{YZ} = 4$; slope of $\overline{WZ} = -\frac{1}{4}$; slope of $\overline{XW} = 4$; the product of the slopes of adjacent sides is -1. Therefore, all angles are rt. angles, and $WXYZ$ is a rectangle.

Also available online

Technology LAB Organizer

Use with the lesson Transforming Linear Functions

Pacing:
Traditional $\frac{1}{2}$ day
Block $\frac{1}{4}$ day

Objective: Use a graphing calculator to explore the family of linear functions.

Materials: graphing calculator

Online Edition
Graphing Calculator, TechKeys

Resources

Technology Lab Activities
Lab Recording Sheet

Teach

Discuss

The order in which the equations are entered into the **Y1** editor will be the same order in which the lines are graphed. Before pressing GRAPH, note if the values of b in the equations are starting out negative or positive, and if they are in ascending or descending order.

Close

Key Concept

Changing b shifts a graph up or down. Changing m turns a graph so it is steeper or less steep.

Assessment

Journal Have students compare and contrast the graphs of $y = x$, $y = x + 10$, and $y = 10x$.

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

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4-10 Technology LAB

Use with Transforming Linear Functions

The Family of Linear Functions

A *family of functions* is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family. You can use a graphing calculator to explore families of functions.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

Learn It Online
Lab Resources Online

Activity

Graph the lines described by $y = x - 2$, $y = x - 1$, $y = x$, $y = x + 1$, $y = x + 2$, $y = x + 3$, and $y = x + 4$. How does the value of b affect the graph described by $y = x + b$?

1. All of the functions are in the form $y = x + b$. Enter them into the **Y=** editor.

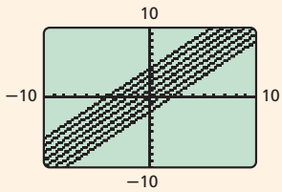
   Y= X,T,θ,n (−) 2 ENTER
   X,T,θ,n (−) 1 ENTER

   and so on.

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k. . . .

2. Press ZOOM and select **6:Zstandard**. Think about the different values of b as you watch the graphs being drawn. Notice that the lines are all parallel.

3. It appears that the value of b in $y = x + b$ shifts the graph up or down—up if b is positive and down if b is negative.



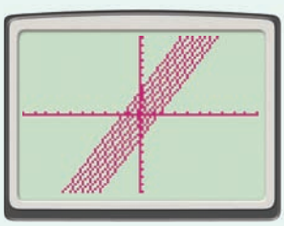
Try This

1. Make a prediction about the lines described by $y = 2x - 3$, $y = 2x - 2$, $y = 2x - 1$, $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, and $y = 2x + 3$. Then graph. Was your prediction correct?
2. Now use your calculator to explore what happens to the graph of $y = mx$ when you change the value of m.
   a. **Make a Prediction** How do you think the lines described by $y = -2x$, $y = -x$, $y = x$, and $y = 2x$ will be related? How will they be alike? How will they be different?
   b. Graph the functions given in part **a**. Was your prediction correct?
   c. How is the effect of m different when m is positive from when m is negative?

2c. When m is pos., the graph slopes upward from left to right and greater values of m mean steeper lines. When m is neg., the graph slopes downward from left to right and greater values of m mean less steep lines.

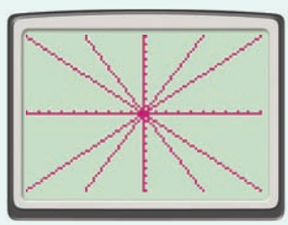
Answers

1. They will all be par. lines.



2a. They will all pass through (0, 0) but have different slopes.

2b.



COMMON CORE

4-10 Transforming Linear Functions

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); ….

Objective
Describe how changing slope and *y*-intercept affect the graph of a linear function.

Vocabulary
family of functions
parent function
transformation
translation
rotation
reflection

Who uses this?
Business owners can use transformations to show the effects of price changes, such as the price of trophy engraving. (See Example 5.)

A **family of functions** is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family because all of their graphs are the same basic shape.

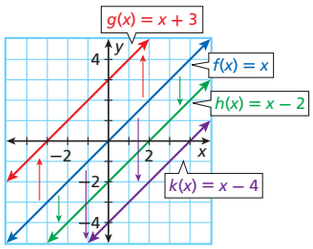
A **parent function** is the most basic function in a family. For linear functions, the parent function is $f(x) = x$.

The graphs of all other linear functions are *transformations* of the graph of the parent function, $f(x) = x$. A **transformation** is a change in position or size of a figure.

There are three types of transformations—*translations, rotations,* and *reflections.*

Look at the four functions and their graphs below.

Remember!
Function notation—$f(x)$, $g(x)$, and so on—can be used in place of *y*.
$y = f(x)$



Notice that all of the lines above are parallel. The slopes are the same but the *y*-intercepts are different.

The graphs of $g(x) = x + 3$, $h(x) = x - 2$, and $k(x) = x - 4$ are vertical *translations* of the graph of the parent function, $f(x) = x$. A **translation** is a type of transformation that moves every point the same distance in the same direction. You can think of a translation as a "slide."

Know it! Note

Vertical Translation of a Linear Function

When the *y*-intercept b is changed in the function $f(x) = mx + b$, the graph is translated vertically.

- If b increases, the graph is translated up.
- If b decreases, the graph is translated down.

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4-10 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objective: Describe how changing slope and *y*-intercept affect the graph of a linear function.

Online Edition
Tutorial Videos

Countdown

Power Presentations with PowerPoint®

Warm Up

Identify the slope and *y*-intercept.

1. $y = x + 4$ $m = 1$; $b = 4$
2. $y = -3x$ $m = -3$; $b = 0$

Compare and contrast the graphs of each pair of equations.

3. $y = 2x + 4$ and $y = 2x - 4$
same slope, par.; diff. intercepts
4. $y = 2x + 4$ and $y = -2x + 4$
same *y*-intercepts; diff. slopes but same steepness

Also available online

Math Humor

Q: Why did the student need directions during math class?

A: He was lost in translation.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Learn It Online
State Resources Online

1 Introduce

EXPLORATION

Transforming Linear Functions

You will need graph paper or a graphing calculator for this Exploration.

1. Graph $y = 2x + 3$.
2. Write a new linear function of the form $y = 2x + b$, where b is any number of your choice. Graph this function on the same coordinate plane as $y = 2x + 3$. How does the graph of the new function compare to that of $y = 2x + 3$? Check students' work.
3. Repeat the process several times, choosing different values for b and graphing the new functions on the same coordinate plane. Be sure to try some values greater than 3 and some less than 3. Check students' work.
4. What happened to the graph of $y = 2x + 3$ when you replaced 3 with a number greater than 3? shifted up
5. What happened to the graph of $y = 2x + 3$ when you replaced 3 with a number less than 3? shifted down

THINK AND DISCUSS

6. **Describe** how you would change $y = -7x + 1$ if you wanted to slide its graph down. Replace 1 with any number less than 1.
7. **Describe** how you would change $y = -7x + 1$ if you wanted to slide its graph up by 5 units. Replace 1 with 6.

1.

Motivate

Students most likely studied translations and reflections of geometric figures in previous grades, but they may recognize only informal names such as *slide*, *turn*, and *flip*. Review these concepts with them. Tell them that graphs of functions are geometric figures too, and that they can also be translated and reflected. Tell students that they will learn how such changes to a graph can be related directly to the equation that describes the function.

Explorations and answers are provided in your online resources.

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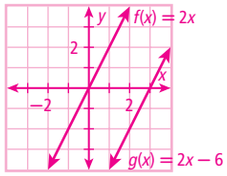
Additional Examples

Example 1

Graph $f(x) = 2x$ and $g(x) = 2x - 6$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

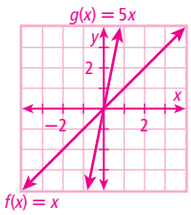
trans. 6 units down



Example 2

Graph $f(x) = x$ and $g(x) = 5x$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

rot. about (0, 0) (steeper)



Also available online

Questioning Strategies

EXAMPLE 1

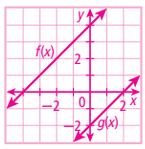
- What part of a linear function indicates which way the line will shift vertically?

EXAMPLE 2

- How can you tell if a transformed function will be steeper or less steep than the original function?

Teaching Tip **Technology** Notice that the slopes appear different on the calculator screen than on the paper graph. By using a square window, you can see the true slope represented on the calculator screen.

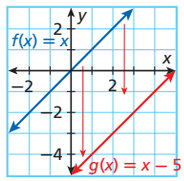
1.

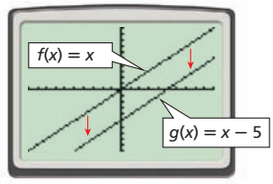


trans. 6 units down

EXAMPLE 1 Translating Linear Functions

Graph $f(x) = x$ and $g(x) = x - 5$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

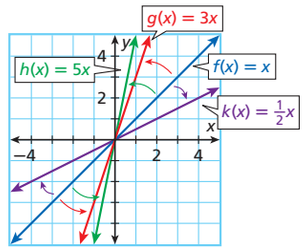




The graph of $g(x) = x - 5$ is the result of translating the graph of $f(x) = x$ 5 units down.

CHECK IT OUT! **1.** Graph $f(x) = x + 4$ and $g(x) = x - 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

The graphs of $g(x) = 3x$, $h(x) = 5x$, and $k(x) = \frac{1}{2}x$ are *rotations* of the graph of $f(x) = x$. A **rotation** is a transformation about a point. You can think of a rotation as a "turn." The y-intercepts are the same, but the slopes are different.



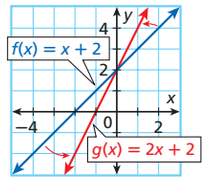
Know it! Note

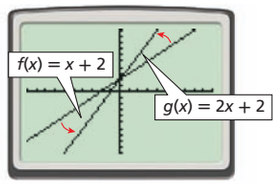
Rotation of a Linear Function

When the slope m is changed in the function $f(x) = mx + b$ it causes a rotation of the graph about the point $(0, b)$, which changes the line's steepness.

EXAMPLE 2 Rotating Linear Functions

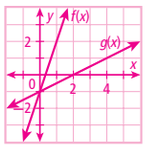
Graph $f(x) = x + 2$ and $g(x) = 2x + 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.





The graph of $g(x) = 2x + 2$ is the result of rotating the graph of $f(x) = x + 2$ about (0, 2). The graph of $g(x)$ is steeper than the graph of $f(x)$.

2.



rot. about (0, −1) (less steep)

CHECK IT OUT! **2.** Graph $f(x) = 3x - 1$ and $g(x) = \frac{1}{2}x - 1$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

2 Teach

Guided Instruction

Review graphing linear equations with an emphasis on slope and y-intercept before beginning this lesson. For each transformation of a linear function, encourage students to ask themselves if the y-intercept and/or the slope will change, and in which direction. This lesson will help prepare students for transformations of nonlinear functions, which are an important concept later in this course as well as in future mathematics courses.

Reaching All Learners

Through Kinesthetic Experience

Give each student three index cards and have them write *translation* on the first, *rotation* on the second, and *reflection* on the third. Write the functions listed below on the board one at a time. Instruct students to hold up the correct card(s) to describe the transformation from $f(x)$ to $g(x)$.

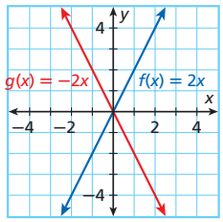
$f(x) = x$, $g(x) = 4x$ rotation

$f(x) = x$, $g(x) = x - 6$ translation

$f(x) = x$, $g(x) = -5x$ rotation

$f(x) = 2x$, $g(x) = -2x$ rotation, reflection

The diagram shows the *reflection* of the graph of $f(x) = 2x$ across the y-axis, producing the graph of $g(x) = -2x$. A **reflection** is a transformation across a line that produces a mirror image. You can think of a reflection as a "flip" over a line.



Know it! Note

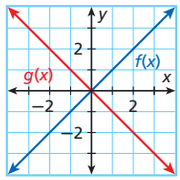
Reflection of a Linear Function

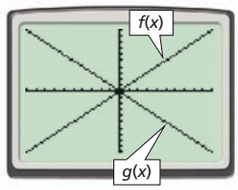
When the slope m is multiplied by -1 in $f(x) = mx + b$, the graph is reflected across the y-axis.

EXAMPLE 3 Reflecting Linear Functions

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

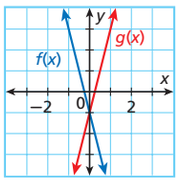
A $f(x) = x$

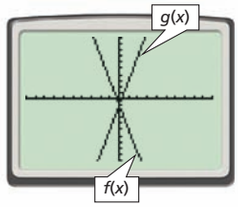




To find $g(x)$, multiply the value of m by -1.
In $f(x) = x$, $m = 1$.
$1(-1) = -1$ *This is the value of m for g(x).*
$g(x) = -x$

B $f(x) = -4x - 1$



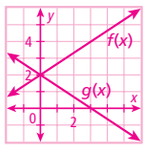


To find $g(x)$, multiply the value of m by -1.
In $f(x) = -4x - 1$, $m = -4$.
$-4(-1) = 4$ *This is the value of m for g(x).*
$g(x) = 4x - 1$

CHECK IT OUT!

3. Graph $f(x) = \frac{2}{3}x + 2$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

3.



$g(x) = -\frac{2}{3}x + 2$

COMMON ERROR ALERT

Students might think that a positive slope will always be steeper than a negative slope because the positive value is greater. To show this is not true, have students graph $f(x) = -6x$ and $g(x) = 2x$. Explain that it is necessary to take the absolute value of each slope before comparing them.

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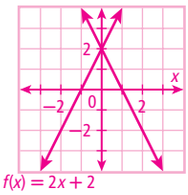
Additional Examples

Example 3

Graph $f(x) = 2x + 2$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.



$g(x) = -2x + 2$

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 3

- How does the slope change in a reflection? How does the y-intercept change?

Teaching Tip

Math Background

Some students may notice that a reflection of a linear function can also be described as a rotation. This is true for linear functions, because both are the result of changing the slope. However, this is not the case for other functions that students will study later in this course and in future courses. Be sure students connect the concepts "multiply by -1" and "reflection."

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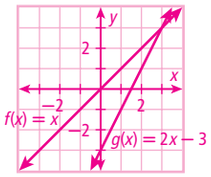
Additional Examples

Example 4

Graph $f(x) = x$ and $g(x) = 2x - 3$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.



rot. about (0, 0) (steeper) and trans. 3 units down

Example 5

A florist charges $25 for a vase plus $4.50 for each flower. The total charge for the vase and flowers is given by the function $f(x) = 4.50x + 25$. How will the graph change if the vase's cost is raised to $35? if the charge per flower is lowered to $3.00?

translated 10 units up; rotated about (0, 25) (less steep)

Also available online

MATHEMATICAL PRACTICES Questioning Strategies

EXAMPLE 4

- Does the order of multiple transformations make a difference on the resulting graph? Explain.

EXAMPLE 5

- Do you think a translation or a rotation will have more effect on the overall cost? Explain your reasoning.

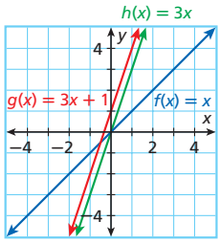
EXAMPLE 4 Multiple Transformations of Linear Functions

Graph $f(x) = x$ and $g(x) = 3x + 1$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

Find transformations of $f(x) = x$ that will result in $g(x) = 3x + 1$:

- Multiply $f(x)$ by 3 to get $h(x) = 3x$. This rotates the graph about $(0, 0)$ and makes it steeper.
- Then add 1 to $h(x)$ to get $g(x) = 3x + 1$. This translates the graph 1 unit up.

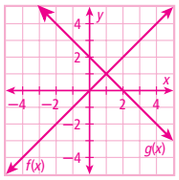
The transformations are a rotation and a translation.



CHECK IT OUT!

4. Graph $f(x) = x$ and $g(x) = -x + 2$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

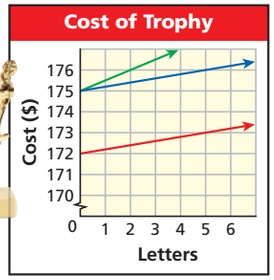
4.



ref. across y-axis, trans. 2 units up

EXAMPLE 5 Business Application

A trophy company charges $175 for a trophy plus $0.20 per letter for the engraving. The total charge for a trophy with x letters is given by the function $f(x) = 0.20x + 175$. How will the graph change if the trophy's cost is lowered to $172? if the charge per letter is raised to $0.50?



$f(x) = 0.20x + 175$ is graphed in blue.

If the trophy's cost is lowered to $172, the new function is $g(x) = 0.20x + 172$. The original graph will be translated 3 units down.

If the charge per letter is raised to $0.50, the new function is $h(x) = 0.50x + 175$. The original graph will be rotated about $(0, 175)$ and become steeper.

CHECK IT OUT!

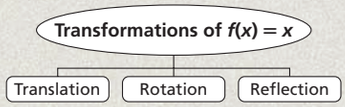
5. What if...? How will the graph change if the charge per letter is lowered to $0.15? if the trophy's cost is raised to $180?

rotated about (0, 175) (less steep); translated 5 units up

MATHEMATICAL PRACTICES THINK AND DISCUSS

1. Describe the graph of $f(x) = x + 3.45$
2. Look at the graphs in Example 5. For each line, is every point on the line a solution in this situation? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a graph of the given transformation of $f(x) = x$, and label it with a possible equation.

Know it! Note



Victoria Smith/HMH

3 Close

Summarize

Ask students to state how each transformation affects the slope and y-intercept.

translation
same slope, diff. y-int.

rotation
diff. slope, same y-int.

reflection
opp. slope, same y-int.

ONGOING ASSESSMENT and INTERVENTION

***Diagnose Before* the Lesson**
Warm Up, TE

***Monitor During* the Lesson**
Check It Out! Exercises, SE
Questioning Strategies, TE

***Assess After* the Lesson**
Lesson Quiz, TE
Alternative Assessment, TE

Answers to *Think and Discuss*

1. translation of $f(x) = x$ up 3.45 units
2. No; there can only be a whole number of letters, so points whose x-coordinates are not whole numbers have no meaning in this situation.
3. See Additional Answers.

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Parent Resources Online

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Changing the value of b in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *reflection*) **translation**
2. Changing the value of m in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *rotation*) **rotation**

Graph $f(x)$ and $g(x)$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE 1

3. $f(x) = x, g(x) = x - 4$
4. $f(x) = x, g(x) = x + 1$
5. $f(x) = x, g(x) = x + 2$
6. $f(x) = x, g(x) = x - 6.5$

SEE EXAMPLE 2

7. $f(x) = x, g(x) = \frac{1}{4}x$
8. $f(x) = \frac{1}{5}x + 3, g(x) = x + 3$
9. $f(x) = 2x - 2, g(x) = 4x - 2$
10. $f(x) = x + 1, g(x) = \frac{1}{2}x + 1$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

SEE EXAMPLE 3

11. $f(x) = -\frac{1}{5}x$ **$g(x) = \frac{1}{5}x$**
12. $f(x) = 2x + 4$ **$g(x) = -2x + 4$**
13. $f(x) = \frac{1}{3}x - 6$ **$g(x) = -\frac{1}{3}x - 6$**
14. $f(x) = 5x - 1$ **$g(x) = -5x - 1$**

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE 4

15. $f(x) = x, g(x) = 2x - 2$
16. $f(x) = x, g(x) = \frac{1}{3}x + 1$
17. $f(x) = -x - 1, g(x) = -4x$
18. $f(x) = -x, g(x) = -\frac{1}{2}x - 3$

SEE EXAMPLE 5

19. **Entertainment** For large parties, a restaurant charges a reservation fee of $25, plus $15 per person. The total charge for a party of x people is $f(x) = 15x + 25$. How will the graph of this function change if the reservation fee is raised to $50? if the per-person charge is lowered to $12?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–25	3
26–27	4
28	5

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

20. $f(x) = x, g(x) = x + \frac{1}{2}$
21. $f(x) = x, g(x) = x - 4$
22. $f(x) = \frac{1}{5}x - 1, g(x) = \frac{1}{10}x - 1$
23. $f(x) = x + 2, g(x) = \frac{2}{3}x + 2$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

24. $f(x) = 6x$ **$g(x) = -6x$**
25. $f(x) = -3x - 2$ **$g(x) = 3x - 2$**

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

26. $f(x) = 2x, g(x) = 4x - 1$
27. $f(x) = -7x + 5, g(x) = -14x$

4-10 Exercises

Assignment Guide

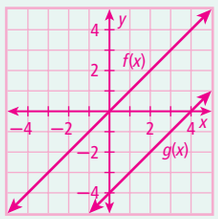
Assign *Guided Practice* exercises as necessary.

If you finished Examples **1–3**
Basic 20–25, 30, 40, 44
Average 20–25, 30, 40, 51
Advanced 20–25, 30, 51, 52

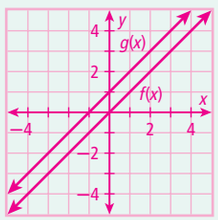
If you finished Examples **1–5**
Basic 20–35, 39–44, 47, 49, 50
Average 20–28, 35–51
Advanced 20–28, 36–52

Homework Quick Check
Quickly check key concepts.
Exercises: 20, 22, 24, 26, 28, 42

Answers

3. 

trans. 4 units down

4. 

trans. 1 unit up

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them. Exercises 19, 28, 38, 45, 48

Construct viable arguments and critique the reasoning of others. Exercises 46–47

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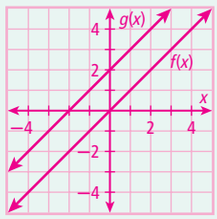
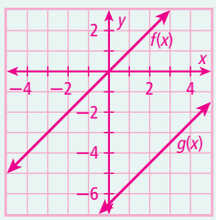
State Resources Online

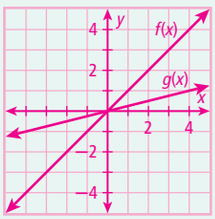
Answers

5. 

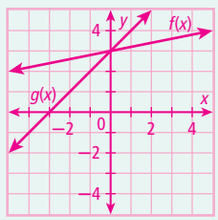
trans. 2 units up

6. 

trans. 6.5 units down

7. 

rot. about (0, 0) (less steep)

8. 

rot. about (0, 3) (steeper)

9, 10. See Additional Answers.

11–14. For graphs, see Additional Answers.

15–23. See Additional Answers.

24, 25. For graphs, see Additional Answers.

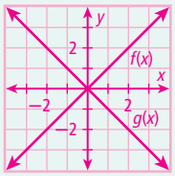
26, 27. See Additional Answers.

MULTI-STEP TEST PREP **Exercise 48** involves a translation of a line. This exercise prepares students for the Multi-Step Test Prep.

Answers

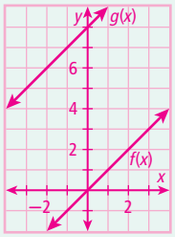
28. trans. 2 units down; rot. about (0, 2) (steeper)

29. ref. across *y*-axis



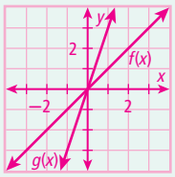
opp. slopes—same steepness but in opp. directions; same *y*-int.

30. trans. 8 units up



same slope but diff. *y*-intercepts

31. rot. about (0, 0) (steeper)



diff. slopes and same *y*-int.

32–34. See Additional Answers.

35–38b. For graphs, see Additional Answers.

38c–44, 47, 48. See Additional Answers.

28. School The number of chaperones on a field trip must include 1 teacher for every 4 students, plus 2 parents total. The function describing the number of chaperones for a trip of x students is $f(x) = \frac{1}{4}x + 2$. How will the graph change if the number of parents is reduced to 0? if the number of teachers is raised to 1 for every 3 students?

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$. Graph $f(x)$ and $g(x)$, and compare the slopes and intercepts.

29. $g(x) = -x$ **30.** $g(x) = x + 8$ **31.** $g(x) = 3x$

32. $g(x) = -\frac{2}{7}x$ **33.** $g(x) = 6x - 3$ **34.** $g(x) = -2x + 1$

Sketch the transformed graph. Then write a function to describe your graph.

35. Rotate the graph of $f(x) = -x + 2$ until it has the same steepness in the opposite direction. $g(x) = x + 2$

$g(x) = -x - 5$

36. Reflect the graph of $f(x) = x - 1$ across the *y*-axis, and then translate it 4 units down.

37. Translate the graph of $f(x) = \frac{1}{6}x - 10$ six units up. $g(x) = \frac{1}{6}x - 4$

38. Hobbies A book club charges a membership fee of $20 and then $12 for each book purchased.

a. Write and graph a function to represent the cost y of membership in the club based on the number of books purchased x. $y = 12x + 20$

b. What if...? Write and graph a second function to represent the cost of membership if the club raises its membership fee to $30. $y = 12x + 30$

c. Describe the relationship between your graphs from parts **a** and **b.**

LINK

Hobbies

Seattle librarian Nancy Pearl started the first citywide reading program, "If All Seattle Reads the Same Book," in 1998. Many cities have since emulated this program, attempting to unite communities by having everyone read the same book at the same time.

45c. Commission changes to 25%; base pay changes to $400.

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$.

39. $g(x) = x - 9$ **40.** $g(x) = -x$ **41.** $g(x) = 5x$

42. $g(x) = -\frac{2}{3}x + 1$ **43.** $g(x) = -2x$ **44.** $g(x) = \frac{1}{5}x$

45. Careers Kelly works as a salesperson. She earns a weekly base salary plus a commission that is a percent of her total sales. Her total weekly pay is described by $f(x) = 0.20x + 300$, where x is total sales in dollars.

a. What is Kelly's weekly base salary? $300

b. What percent of total sales does Kelly receive as commission? 20%

c. What if...? What is the change in Kelly's salary plan if the weekly pay function changes to $g(x) = 0.25x + 300$? to $h(x) = 0.2x + 400$?

46. Critical Thinking To transform the graph of $f(x) = x$ into the graph of $g(x) = -x$, you can reflect the graph of $f(x)$ across the *y*-axis. Find another transformation that will have the same result. Possible answer: ref. across *x*-axis

47. Write About It Describe how a reflection across the *y*-axis affects each point on a graph. Give an example to illustrate your answer.

MULTI-STEP TEST PREP

48. a. Maria is walking from school to the softball field at a rate of 3 feet per second. Write a rule that gives her distance from school (in feet) as a function of time (in seconds). Then graph.

b. Give a real-world situation that could be described by a line parallel to the one in part **a.**

c. What does the *y*-intercept represent in each of these situations?

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PRACTICE A

PRACTICE C

PRACTICE B

Graph $f(x)$ and $g(x)$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

1. $f(x) = x$; $g(x) = x + 3$
translation 3 units up

2. $f(x) = \frac{1}{3}x - 4$; $g(x) = \frac{1}{4}x - 4$
rotation (less steep) about (0, –4)

3. $f(x) = x$; $g(x) = 2x - 5$
rotation (steeper) about (0, 0) and translation 5 units down

4. Graph $f(x) = -3x + 1$. Then reflect the graph of $f(x)$ across the *y*-axis. Write a function $g(x)$ to describe the new graph.
$g(x) = 3x + 1$

5. The cost of hosting a party at a horse farm is a flat fee of $250, plus $5 per person. The total charge for a party of x people is $f(x) = 5x + 250$. How will the graph of this function change if the flat fee is lowered to $200? if the per-person rate is raised to $8?
The graph will be translated 50 units down. The graph will be rotated about (0, 250) and become steeper.

READING STRATEGIES

The table shows three types of transformations.

Transformation	Translation	Rotation	Reflection
Description	When b in $y = mx + b$ increases (decreases), the line is translated up (down).	When $\|m\|$ in $y = mx + b$ increases (decreases), the line becomes more steep (less steep).	When m in $y = mx + b$ is multiplied by -1, the line is reflected across the *y*-axis.
Graph			
Equations	$f(x) = -3x + 2$ $g(x) = -3x - 3$	$f(x) = -3x + 2$ $g(x) = -1x + 2$	$f(x) = -3x + 2$ $g(x) = 3x + 2$

Answer the following.

1. Look at the graph of the translation. How many vertical units is $f(x)$ shifted down to $g(x)$? 5
Look at the related functions. Subtract the *y*-intercept of $g(x)$ from the *y*-intercept of $f(x)$. 2 – (–3) = 5

2. Look at the graph of the rotation. Which graph is steeper? $f(x)$
Look at the slopes of the related functions. For which function is the absolute value of the slope greater? $f(x)$

3. Look at the graph of the reflection. Which axis is a line of symmetry? *y*-axis
Look at the slopes of the related functions. Are they the same or opposite? opposite

Describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

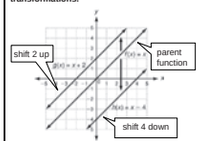
4. $f(x) = 4x - 3$, $g(x) = 2x - 3$ rotation (less steep) about (0, –3)
5. $f(x) = -2x + 2$, $g(x) = 2x + 2$ reflection about *y*-axis
6. $f(x) = x + 2$, $g(x) = x + 8$ translation 6 units up

RETEACH

The function $f(x) = x$ is called the parent function for the family of linear functions.

Changing the *y*-intercept will shift the line up or down. This is called a **translation.**

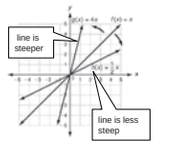
Graph $f(x) = x$, $g(x) = x + 2$, and $h(x) = x - 4$. Describe the transformations.



The *y*-intercept of $g(x)$ is 2 so the graph shifts up 2 units from $f(x)$. The *y*-intercept of $h(x)$ is –4 so the graph shifts down 4 units from $f(x)$.

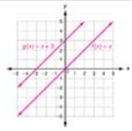
Changing the slope of a line will change the steepness of the line. This is called a **rotation.**

Graph $f(x) = x$, $g(x) = 4x$, and $h(x) = \frac{1}{2}x$. Describe the transformations.



The slope of $g(x)$ is 4 so the line is steeper than $f(x)$ which has a slope of 1. The slope of $h(x)$ is $\frac{1}{2}$ and it is less steep.

1. Graph $f(x) = x$ and $g(x) = x + 3$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.
translation 3 units up



Describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

2. $f(x) = x$; $g(x) = \frac{1}{3}x$ rotation (less steep)
3. $f(x) = x$; $g(x) = x - 5$ translation 5 units down
4. $f(x) = x$; $g(x) = 6x$ rotation (steeper)

TEST PREP

49. Which best describes the effect on $f(x) = 2x - 5$ if the slope changes to 10?

- (A) Its graph becomes less steep.
- (B) Its graph moves 15 units up.
- (C) Its graph makes 10 complete rotations.
- (D) The x-intercept becomes $\frac{1}{2}$.

50. Given $f(x) = 22x - 182$, which does NOT describe the effect of increasing the y-intercept by 182?

- (F) The new line passes through the origin.
- (G) The new x-intercept is 0.
- (H) The new line is parallel to the original.
- (J) The new line is steeper than the original.

CHALLENGE AND EXTEND

51. You have seen that the graph of $g(x) = x + 3$ is the result of translating the graph of $f(x) = x$ three units up. However, you can also think of this as a *horizontal* translation—that is, a translation left or right. Graph $g(x) = x + 3$. Describe the horizontal translation of the graph of $f(x) = x$ to get the graph of $g(x) = x + 3$.

trans. 3 units to the left

52. If $c > 0$, how can you describe the translation that transforms the graph of $f(x) = x$ into the graph of $g(x) = x + c$ as a horizontal translation? $g(x) = x - c$ as a horizontal translation? trans. c units to the left; trans. c units to the right

PROBLEM SOLVING

Write the correct answer.

1. The number of camp counselors at a day camp must include 1 counselor for every 8 campers, plus 3 camp directors. The function describing the number of counselors is $f(x) = \frac{1}{8}x + 3$ where x is the number of campers. How will the graph change if the number of camp directors is reduced to 2?
   translation 1 unit down
2. A city water service has a base cost of $12 per month plus $1.50 per hundred cubic feet (HCF) of water. Write a function $f(x)$ to represent the cost of water as a function of x, amount used. Then write a second function $g(x)$ to represent the cost if the rate rises to $1.60 per HCF.
   $f(x) = 1.50x + 12$; $g(x) = 1.60x + 12$;
   How would the graph of $g(x)$ compare to the graph of $f(x)$?
   it would be rotated about (0, 12), steeper.
3. Owen earns a base salary plus a commission that is a percent of his total sales. His total weekly pay is described by $f(x) = 0.15x + 325$, where x is his total sales in dollars. What is the change in Owen's salary plan if his total weekly pay function changes to $g(x) = 0.20x + 325$?
   His commission is raised to 20%.

An attorney charges $250 per hour. The graph represents the cost of the attorney as a function of time. Select the best answer.

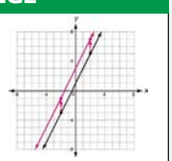
4. When a traveling fee is added to the attorney's rate for cases outside the city limits, the graph is translated up 50 units. What function $h(x)$ would describe the attorney's rate with the traveling fee?
   A $h(x) = 250x - 50$
   B $h(x) = 250x + 50$
   C $h(x) = 200x$
   D $h(x) = 300x$
5. The attorney's paralegal has an hourly rate of $150. How would you transform the graph of $f(x)$ into a graph for the paralegal's rate?
   F Reflect it over the y-axis.
   G Translate it down 100 units.
   H Translate it to the left 100 units.
   J Rotate it clockwise about (0, 0).

6. Which hourly rate would NOT make the attorney's graph steeper?
   A $225 C $300
   B $275 D $325

CHALLENGE

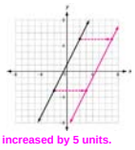
For 1–5, use the two points and line graphed at right.

1. Write an equation in point-slope form for the line. Circle the point that you used as (x_1, y_1).
   $y + 3 = 2(x + 2)$ OR $y - 5 = 2(x - 2)$
2. Translate the line up 2 units by doing the following:
   a. Translate each of the two points up 2 units and plot a new point. Each translated point is an **image** of the original point. Circle the image of the point that you used in problem 1.
   b. Connect the two new points with a line.
3. Use the coordinates of the image that you circled to write an equation in point-slope form for the new line. $y + 1 = 2(x + 2)$ OR $y - 7 = 2(x - 2)$
4. Compare your equations for problems 1 and 3. What indicates that there was a vertical translation up 2 units?
   In the form $y - y_1 = m(x - x_1)$, the value of y_1, increased by 2 units.
5. Suppose that the *original* line is translated down 6 units. Without graphing the transformation, write an equation in point-slope form for the new line. $y + 9 = 2(x + 2)$ OR $y + 1 = 2(x - 2)$



For 6–10, use the graph at right.

6. Translate the line right 5 units. (Use a process similar to problem 2.) Again, circle the image of the point that you used in problem 1.
7. Use the coordinates of the image that you circled to write an equation in point-slope form for the new line.
   $y + 3 = 2(x - 3)$ OR $y - 5 = 2(x - 7)$
8. Compare your equations for problems 1 and 7. What indicates a horizontal translation right 5 units?
   In the form $y - y_1 = m(x - x_1)$, the value of x_1, increased by 5 units.
9. Suppose that the *original* line was translated left 3 units. Without graphing the transformation, write an equation in point-slope form for the new line. $y + 3 = 2(x + 5)$ OR $y - 5 = 2(x + 1)$
10. The equation $y - 3 = -4(x + 2)$ is transformed into $y + 1 = -4(x - 7)$. Describe the transformations from the graph of the first line to the graph of the second.
    Translated down 4 units and right 9 units.



TEST PREP DOCTOR

In **Exercise 49,** the answer must be **A** or **D** because changing slope affects steepness and possibly the x-intercept. Choice **A** can be eliminated because the increase in the absolute value of the slope indicates that the line will be steeper.

Answers

51. For graph, see Additional Answers.

Journal

Have students explain the changes in the slope and y-intercept for a rotation, a translation, and a reflection.

ALTERNATIVE ASSESSMENT

Have students write possible functions representing transformations according to the following directions:

1. Rotate the graph of $f(x) = x$.
2. Translate the graph of $f(x) = x - 4$ up.
3. Reflect the graph of $f(x) = -3x$.

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4-10 Lesson Quiz

Describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

1. $f(x) = 4x$, $g(x) = x$
   rot. about (0, 0) (less steep)
2. $f(x) = x - 1$, $g(x) = x + 6$
   trans. 7 units up
3. $f(x) = \frac{1}{5}x$, $g(x) = 2x$
   rot. about (0, 0) (steeper)
4. $f(x) = 5x$, $g(x) = -5x$
   ref. over y-axis, rot. about (0, 0)
5. $f(x) = x$, $g(x) = x - 4$
   trans. 4 units down
6. $f(x) = -3x$, $g(x) = -x + 1$
   rot. about (0, 0) (less steep), trans. 1 unit up
7. A cashier gets a $50 bonus for working on a holiday plus $9/h. The total holiday salary is given by the function $f(x) = 9x + 50$. How will the graph change if the bonus is raised to $75? if the hourly rate is raised to $12/h?
   translated 25 units up; rotated about (0, 50) (steeper)

Also available online

SECTION 4B

MULTI-STEP TEST PREP

Organizer

Objective: Assess students' ability to apply concepts and skills a real-world format.

Online Edition

Answers

3.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

Learn It Online
State Resources Online

CHAPTER 4

SECTION 4B

MULTI-STEP TEST PREP

MATHEMATICAL PRACTICES **Make sense of problems and persevere in solving them.**

Using Linear Functions

Take a Walk! All intersections in Durango, Colorado, have crossing signals with timers. Once the signal changes to walk, the timer begins at 28 seconds and counts down to show how much time pedestrians have to cross the street.

1. Pauline counted her steps as she crossed the street. She counted 15 steps with 19 seconds remaining. When she reached the opposite side of the street, she had counted a total of 30 steps and had 10 seconds remaining. Copy and complete the table below using these values.

Time Remaining (s)	28	19	10
Steps	0	15	30

2. Find the average rate of change for Pauline's walk. $-\frac{5}{3}$, or $-1\frac{2}{3}$, steps/s

3. Sketch a graph of the points in the table, or plot them on your graphing calculator.

4. Find an equation that describes the line through the points. $y = -\frac{5}{3}x + \frac{140}{3}$

5. How would the graph change if Pauline increased her speed? What if she decreased her speed? The line would become steeper; the line would become less steep.

(tl), © Brand X Pictures; (b), © Tim Pannell/CORBIS; (tr), Andy Christiansen/HMH

INTERVENTION

Scaffolding Questions

1–2. What are the independent and dependent variables? ind.: time remaining; dep.: steps

3–4. Will the points go upward or downward from left to right? Explain. Downward; as time remaining increases, number of steps taken decreases. What does the slope represent? steps per time remaining

5a. How can you find the equation for the relationship? Possible answer: Use point-slope form.

5b. In terms of the number of steps, what does it mean to increase or decrease speed? If speed increases, the number of steps increases; if speed decreases, the number of steps decreases.

Extension

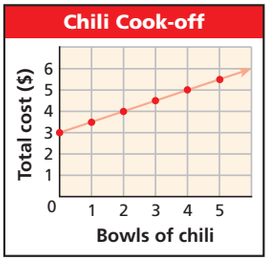
Change the given situation in such a way that the graph is a vertical translation of your graph in **Problem 3.** Possible answer: Pauline walks at the same speed, but the dist. across the street is longer.

Quiz for Lessons 4-6 Through 4-10

4-6 Slope-Intercept Form

Write each equation in slope-intercept form. Then graph the line described by the equation.

1. $2x + y = 5$ $y = -2x + 5$
2. $2x - 6y = 6$ $y = \frac{1}{3}x - 1$
3. $3x + y = 3x - 4$ $y = -4$
4. **Entertainment** At a chili cook-off, people pay a $3.00 entrance fee and $0.50 for each bowl of chili they taste. The graph shows the total cost per person as a function of the number of bowls of chili tasted.



   a. Write an equation that represents the total cost per person as a function of the number of bowls of chili tasted. a. $y = 0.5x + 3$

   b. Identify the slope and y-intercept and describe their meanings. slope: 0.5; cost per bowl of chili; y-int.: 3; entrance fee

4-7 Point-Slope Form

Graph the line with the given slope that contains the given point.

5. slope $= -3$; $(0, 3)$
6. slope $= -\frac{2}{3}$; $(-3, 5)$
7. slope $= 2$; $(-3, -1)$

Write an equation in slope-intercept form for the line through the two points.

8. $(3, 1)$ and $(4, 3)$ $y = 2x - 5$
9. $(-1, -1)$ and $(1, 7)$ $y = 4x + 3$
10. $(1, -4)$ and $(-2, 5)$ $y = -3x - 1$

4-8 Line of Best Fit

11. A street vendor noted the daily high temperature and the number of ice cream cones she sold each day for one week. Find an equation for a line of best fit. How well does the line represent the data? $y \approx 0.74x - 27$; extremely well, $(r \approx 0.99)$

Temperature (°F)	80	73	65	90	96	100	82
Ice Cream Cones Sold	32	27	22	38	45	48	35

4-9 Slopes of Parallel and Perpendicular Lines

Identify which lines are parallel.

12. $y = -2x$; $y = 2x + 1$; $y = 2x$; $y = 2(x + 5)$ $y = 2x + 1$, $y = 2x$, and $y = 2(x + 5)$
13. $-3y = x$; $y = -\frac{1}{3}x + 1$; $y = -3x$; $y + 2 = x + 4$ $-3y = x$ and $y = -\frac{1}{3}x + 1$

Identify which lines are perpendicular.

14. $y = -4x - 1$; $y = \frac{1}{4}x$; $y = 4x - 6$; $x = -4$
15. $y = -\frac{3}{4}x$; $y = \frac{3}{4}x - 3$; $y = \frac{4}{3}x$; $y = 4$; $x = 3$

4-10 Transforming Linear Functions

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

16. $f(x) = 5x$, $g(x) = -5x$ ref. across y-axis; rot. about (0, 0)
17. $f(x) = \frac{1}{2}x - 1$, $g(x) = \frac{1}{2}x + 4$ trans. 5 units up

READY TO GO ON? SECTION 4B

Organizer

Objective: Assess students' mastery of concepts and skills in this section.

Resources

Assessment Resources
Section Quiz

INTERVENTION

Resources

***Ready to Go On? Intervention and Enrichment* Worksheets**

***Ready to Go On?* Online**

Answers

1–3. For graphs, see Additional Answers.

5–7, 14, 15. See Additional Answers.

16–17. For graphs, see Additional Answers.

READY TO GO ON?
Diagnose and Prescribe

NO INTERVENE

YES ENRICH

Ready to Go On? Intervention	*Ready to Go On? Intervention*, Section 4B		
	Worksheets	Online	
Lesson 4-6	4-6 Intervention	Activity 4-6	Diagnose and Prescribe Online
Lesson 4-7	4-7 Intervention	Activity 4-7	
Lesson 4-8	4-8 Intervention	Activity 4-8	
Lesson 4-9	4-9 Intervention	Activity 4-9	
Lesson 4-10	4-10 Intervention	Activity 4-10	

***Ready to Go On? Enrichment*, Section 4B**
Worksheets
Online

4 Organizer

Pacing: Traditional 1 day
Block $\frac{1}{2}$ day

Objectives: Graph absolute-value functions.

Identify characteristics of absolute-value functions and their graphs.

Online Edition

Using the Extension

In this Chapter, students learn to graph a linear function and identify characteristics of the graph, such as its slope and x- and y-intercepts. In this extension, students learn to graph an absolute-value function and identify characteristics of the graph, such as its vertex and axis of symmetry.

Teaching Tip **Kinesthetic** Have students graph $f(x) = x$ on a piece of tracing paper. Tell them to fold up the bottom of the graph along the x-axis so the bottom of the graph meets the top. Say that this is the graph of $f(x) = |x|$; all the y-values that were negative are now positive.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

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COMMON CORE

EXTENSION

Absolute-Value Functions

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. … *Also* **CC.9-12.F.IF.7***

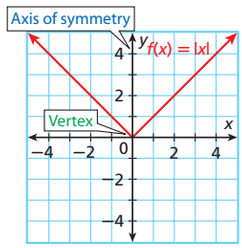
Objectives
Graph absolute-value functions.

Identify characteristics of absolute-value functions and their graphs.

Vocabulary
absolute-value function
axis of symmetry
vertex

An **absolute-value function** is a function whose rule contains an absolute-value expression. To graph an absolute-value function, choose several values of x and generate some ordered pairs.

x	$f(x) = \|x\|$
−2	2
−1	1
0	0
1	1
2	2



Absolute-value graphs are composed of two linear pieces. The **axis of symmetry** is the line that divides the graph into two congruent halves. The **vertex** is the "corner" point on the graph.

From the table and the graph of $f(x) = |x|$, you can tell that
- the **axis of symmetry** is the y-axis $(x = 0)$.
- the **vertex** is $(0, 0)$.
- the domain (x-values) is the set of all real numbers.
- the range (y-values) is described by $y \geq 0$.
- the x-intercept and the y-intercept are both 0.
- the slope of the left linear piece is -1 and the slope of the right linear piece is 1.

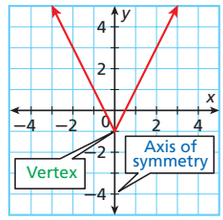
EXAMPLE 1 **Absolute-Value Functions**

Graph $f(x) = |2x| - 1$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

Choose positive, negative, and zero values for x, and find ordered pairs.

x	−2	−1	0	1	2
$f(x) = \|2x\| - 1$	3	1	−1	1	3

Plot the ordered pairs and connect them.



From the table and the graph, you can tell that
- the **axis of symmetry** is the y-axis $(x = 0)$.
- the **vertex** is $(0, -1)$.
- the x-intercepts are $\frac{1}{2}$ and $-\frac{1}{2}$.
- the y-intercept is -1.
- the domain is all real numbers.
- the range is described by $y \geq -1$.
- the slope of the left piece is -2 and the slope of the right piece is 2.

1 Introduce

Motivate

Ask students to do the following:

Find the range for $f(x) = x$ if D: $\{-2, -1, 0, 1, 2\}$. R: $\{-2, -1, 0, 1, 2\}$

Find the range for $f(x) = |x|$ if D: $\{-2, -1, 0, 1, 2\}$. R: $\{2, 1, 0, 1, 2\}$

How are the ranges similar? Both contain 0, 1, 2.

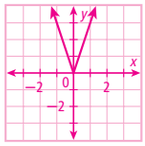
How are the ranges different? The first contains -2 and -1; the second does not.

2 Teach

Guided Instruction

When graphing absolute-value functions by generating ordered pairs, encourage students to substitute zero, positive, and negative values for x. Point out that the absolute-value functions in this extension are translations of the parent function $f(x) = |x|$.

Teaching Tip **Visual** Graphs of absolute-value functions will have a V-shape.

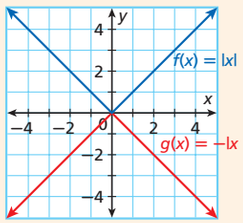
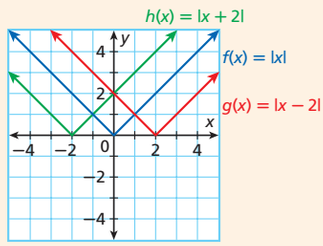
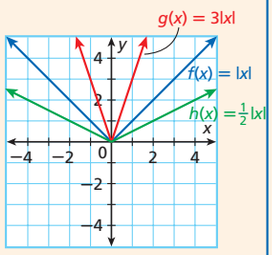
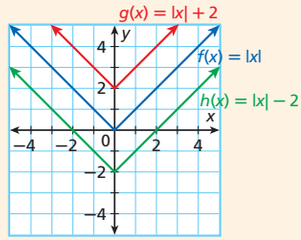


CHECK IT OUT! **1.** Graph $f(x) = 3|x|$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

axis of symm.: $x = 0$; vertex: $(0, 0)$; x-int.: 0; y-int.: 0; D: all real numbers; R: $y \geq 0$; left slope: -3; right slope: 3

The parent function for absolute-value functions is $f(x) = |x|$. The graphs of all other absolute-value functions are transformations of the graph of $f(x) = |x|$.

For $f(x) = a|x - b| + c$

WORDS	EXAMPLES	WORDS	EXAMPLES
Opening Direction • If $a > 0$, the graph opens upward. • If $a < 0$, the graph opens downward.		**Horizontal Translation** • If $b > 0$, the graph is translated b units right from $f(x) = \|x\|$. • If $b < 0$, the graph is translated b units left from $f(x) = \|x\|$.	
Width • If $\|a\| > 1$, the graph is narrower than the graph of $f(x) = \|x\|$. • If $\|a\| < 1$, the graph is wider than the graph of $f(x) = \|x\|$. The slopes of the two linear pieces are a and $-a$.		**Vertical Translation** • If $c > 0$, the graph is translated c units up from $f(x) = \|x\|$. • If $c < 0$, the graph is translated c units down from $f(x) = \|x\|$.	

EXAMPLE 2 Transforming Absolute-Value Functions

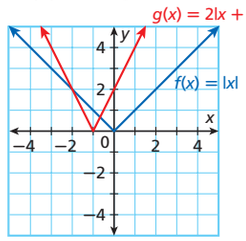
Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

A $g(x) = 2|x + 1|$

Identify a, b, and c.

$g(x) = 2|x + 1| = 2|x - (-1)| + 0$

- $a = 2$: graph is narrower
- $b = -1$: translated 1 unit left
- $c = 0$: no vertical translation



COMMON ERROR ALERT

Students may assume that the range in **Example 1** should contain only positive values. Show students that the substitution of any point between $-\frac{1}{2}$ and $+\frac{1}{2}$ will yield a negative number for this function. Point out how this differs from **Check it Out 1,** where there is no constant term.

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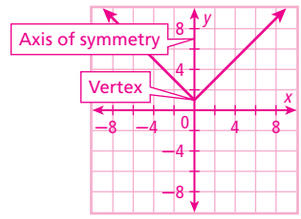
Additional Examples

Example 1

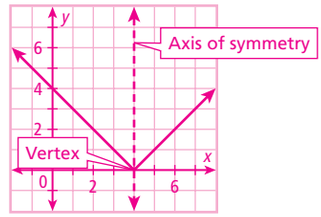
Graph each absolute-value function and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

A. $f(x) = |x| + 1$



axis of symmetry: $x = 0$; vertex: $(0, 1)$; x-int.: none; y-int.: 1; D: all real numbers; R: $y \geq 1$; left slope: -1; right slope: 1

B. $f(x) = |x - 4|$



axis of symmetry: $x = 4$; vertex: $(4, 0)$; x-int.: 4; y-int.: 4; D: all real numbers; R: $y \geq 0$; left slope: -1; right slope: 1

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 1

- How many x- and y-intercepts are possible for an absolute-value function? Explain.
- How can you determine the axis of symmetry simply by looking at the function rule?

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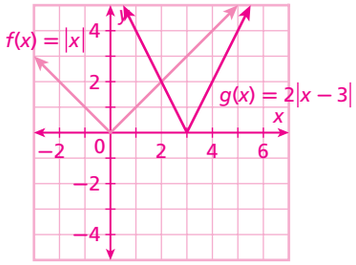
Additional Examples

Example 2

Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

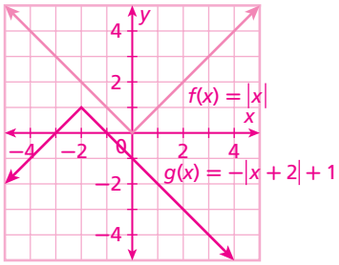
A. $g(x) = 2|x - 3|$

narrowed and trans. 3 units right



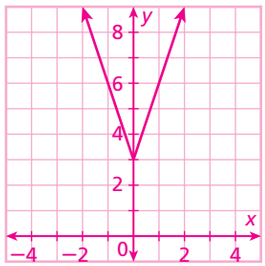
B. $g(x) = -|x + 2| + 1$

refl. across x-axis, trans. 2 units left and 1 unit up



Example 3

Graph $f(x) = |3x| + 3$. Identify the vertex and give the minimum or maximum value of the function.



vertex: (0, 3); minimum: 3

Also available online

Questioning Strategies

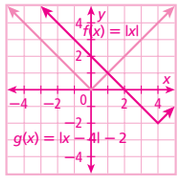
EXAMPLE 2

- How will the graphs of an absolute-value function and its transformation look on the same coordinate plane?

EXAMPLE 3

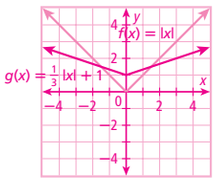
- Is the vertex of an absolute-value function always a minimum or maximum point? Explain.

2a.



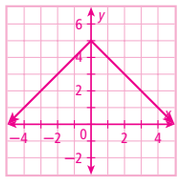
translated 4 units right and 2 units down

2b.



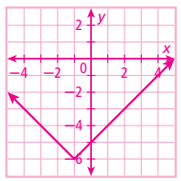
widened and translated 1 unit up

3a.



vertex: (0, 5); maximum: 5

3b.



vertex: (−1, −6); minimum: −6

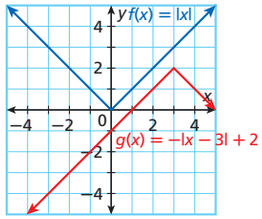
Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

B $g(x) = -|x - 3| + 2$

Identify a, b, and c.

$g(x) = -|x - 3| + 2 = -1|x - 3| + 2$

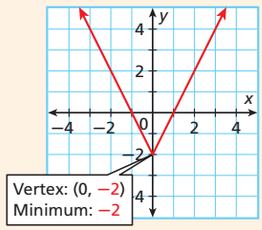
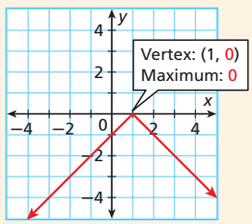
- $a = -1$: graph opens downward and width is unchanged
- $b = 3$: translated 3 units right
- $c = 2$: translated 2 units up



CHECK IT OUT! **Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.**

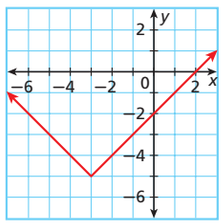
2a. $g(x) = |x - 4| - 2$ **2b.** $g(x) = \frac{1}{3}|x| + 1$

Minimum and Maximum Values of Absolute-Value Functions

WORDS	If the graph of an absolute-value function opens upward, the y-value of the vertex is the **minimum** value of the function.	If the graph of an absolute-value function opens downward, the y-value of the vertex is the **maximum** value of the function.
EXAMPLES	$f(x) = 2\|x\| - 2$ 	$f(x) = -\|x - 1\|$ 

EXAMPLE 3 Identifying the Minimum or Maximum

Graph $f(x) = |x + 3| - 5$. Identify the vertex and give the minimum or maximum value of the function.



The vertex is (−3, −5).

The graph opens upward, so the function has a minimum.

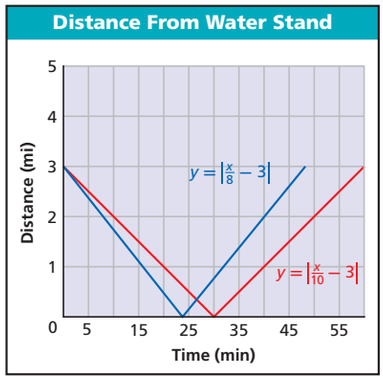
The minimum is −5.

CHECK IT OUT! **Graph each absolute-value function. Identify the vertex and give the minimum or maximum value of the function.**

3a. $f(x) = -|x| + 5$ **3b.** $f(x) = |x + 1| - 6$

EXAMPLE 4 ***Sports Application***

In a charity race, a water stand for the runners is halfway between the start and finish lines. The function $y = \left|\frac{x}{8} - 3\right|$ models Riley's distance y in miles from the water stand x minutes into the race.



A **How long is the race?**

$y = \left|\frac{x}{8} - 3\right|$ is graphed in blue.

At the start of the race ($x = 0$), Riley is 3 mi from the water stand. The water stand is halfway between the start and finish lines, so the race is 6 mi.

B **How much time does it take Riley to reach the water stand?**

When Riley reaches the water stand, $y = 0$. This happens when $x = 24$. It takes Riley 24 min to reach the water stand.

C **The function $y = \left|\frac{x}{10} - 3\right|$ models Dean's distance from the water stand during the same race. Compare Dean's graph to Riley's graph. What can you conclude about Dean's speed?**

$y = \left|\frac{x}{10} - 3\right|$ is graphed in red. Both graphs start at the same point, but Dean's graph is translated to the right. It takes him more time to reach the water stand and to finish the race. Therefore, he is running more slowly than Riley.

CHECK IT OUT! **4.** How would the graph be different for someone who runs faster than Riley? **shifted left and linear pieces would be steeper**

EXTENSION Exercises

Graph each absolute-value function and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece. Identify the maximum or minimum.

1. $f(x) = |x| + 3$ **2.** $f(x) = |x + 3|$ **3.** $f(x) = \frac{1}{2}|x|$ **4.** $f(x) = |x - 3|$

Tell whether each statement is sometimes, always, or never true.

5. The absolute value of a number is negative. **never**

6. An absolute-value function has an x-intercept. **sometimes**

7. An absolute-value function has two y-intercepts. **never**

Write a function to describe each of the following.

8. The graph of $f(x) = |x|$ is translated 2 units right. **$g(x) = |x - 2|$**

9. The graph of $f(x) = |x|$ is narrowed and reflected across the x-axis. **Possible answer: $g(x) = -|4x|$**

10. **Critical Thinking** Suppose that an absolute-value function has no x-intercepts. What can you say about the function rule?
Possible answer: It is in the form $|x| + c$, where $c > 0$.

3 Close

Summarize

Ask students to give the axis of symmetry and vertex for the following functions.

1. $f(x) = |x - 5|$
axis of symmetry: $x = 5$; vertex: $(5, 0)$

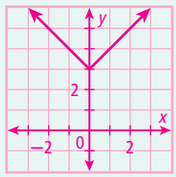
2. $f(x) = |x| - 5$
axis of symmetry: $x = 0$; vertex: $(0, -5)$

3. $f(x) = |x| + 4$
axis of symmetry: $x = 0$; vertex: $(0, 4)$

4. $f(x) = |x + 4|$
axis of symmetry: $x = -4$; vertex: $(-4, 0)$

Answers

1. axis of symmetry: $x = 0$; vertex: $(0, 3)$; x-int.: none; y-int.: 3; D: all real numbers; R: $y \geq 3$; left slope: -1; right slope: 1; minimum: 3



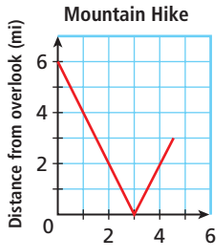
2–4. See Additional Answers.

Power Presentations with PowerPoint®

Additional Examples

Example 4

Rosa and Tony are hiking a mountain trail. Two-thirds of the way up the trail is a scenic overlook. The function $y = |2x - 6|$, graphed below, models Rosa's and Tony's distance y in miles from the overlook after hiking for x hours.



A. How long is the trail? 9 mi

B. How long does it take Rosa and Tony to reach the overlook? 3 h

Also available online

MATHEMATICAL PRACTICES **Questioning Strategies**

EXAMPLE 4

- What are the domain and range of the functions?

Organizer

Objective: Help students organize and review key concepts and skills presented in this chapter.

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Answers

1. translation; rotation; reflection
2. y-intercept
3. slope; y-intercept
4. No; a constant change of $+2$ in x corresponds to different changes in y.
5. Yes; a constant change of $+1$ in x corresponds to a constant change of $+2$ in y.
6. Yes; a constant change of $+1$ in x corresponds to a constant change of -2 in y.
7. No; a constant change of -1 in y corresponds to different changes in x.
8. $5x + y = 1$; $A = 5$; $B = 1$; $C = 1$
9. $x + 6y = -2$; $A = 1$; $B = 6$; $C = -2$
10. $7x - 4y = 0$; $A = 7$, $B = -4$, $C = 0$
11. $y = 9$; $A = 0$; $B = 1$; $C = 9$

CHAPTER 4 Study Guide: Review

Vocabulary

constant of variation	**linear regression**	**rotation**
correlation coefficient	**parallel lines**	**run**
direct variation	**parent function**	**slope**
family of functions	**perpendicular lines**	**transformation**
least-squares line	**rate of change**	**translation**
line of best fit	**reflection**	***x*-intercept**
linear equation	**residual**	***y*-intercept**
linear function	**rise**	

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A(n) ___?___ is a "slide," a(n) ___?___ is a "turn," and a(n) ___?___ is a "flip."
2. The x-coordinate of the point that contains the ___?___ is always 0.
3. In the equation $y = mx + b$, the value of m is the ___?___, and the value of b is the ___?___.

4-1 Identifying Linear Functions

EXAMPLES

Tell whether each function is linear. If so, graph the function.

■ $y = -3x + 2$

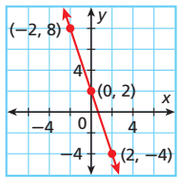
$y = -3x + 2$
$+3x \quad +3x$
$3x + y = 2$

Write the equation in standard form.

This is a linear function.

Generate ordered pairs.

x	$y = -3x + 2$	(x, y)
-2	$y = -3(-2) + 2 = 8$	$(-2, 8)$
x	$y = -3(0) + 2 = 2$	$(0, 2)$
x	$y = -3(2) + 2 = -4$	$(2, -4)$



Plot the points and connect them with a straight line.

■ $y = 2x^3$

This is not a linear function because x has an exponent other than 1.

EXERCISES

Tell whether the given ordered pairs satisfy a linear function. Explain.

4.

x	y
-3	3
-1	1
1	1
3	3

5.

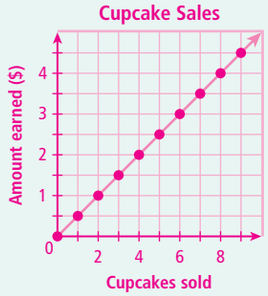
x	y
0	-3
1	-1
2	1
3	3

6. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$
7. $\{(1, 7), (3, 6), (6, 5), (9, 4), (13, 3)\}$

Each equation below is linear. Write each equation in standard form and give the values of *A*, *B*, and *C*.

8. $y = -5x + 1$
9. $\frac{x + 2}{2} = -3y$
10. $4y = 7x$
11. $9 = y$
12. Helene is selling cupcakes for \$0.50 each. The function $f(x) = 0.5x$ gives the total amount of money Helene makes after selling x cupcakes. Graph this function and give its domain and range.

12.



D: whole numbers;
R: nonneg. multiples of 0.5

4-2 Using Intercepts

EXAMPLE

■ **Find the x- and y-intercepts of $2x + 5y = 10$.**

Let $y = 0$.

$2x + 5(0) = 10$

$2x + 0 = 10$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

The x-intercept is 5.

Let $x = 0$.

$2(0) + 5y = 10$

$0 + 5y = 10$

$5y = 10$

$\frac{5y}{5} = \frac{10}{5}$

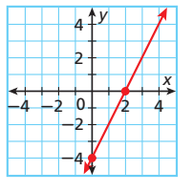
$y = 2$

The y-intercept is 2.

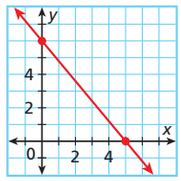
EXERCISES

Find the x- and y-intercepts.

13.



14.



15. $3x - y = 9$

16. $-2x + y = 1$

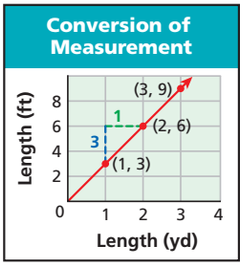
17. $-x + 6y = 18$

18. $3x - 4y = 1$

4-3 Rate of Change and Slope

EXAMPLE

■ **Find the slope of the line.**



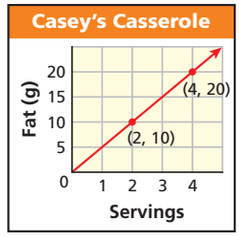
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3}{1} = 3$$

EXERCISES

19. Graph the data and show the rates of change.

Time (s)	Distance (ft)
0	0
1	16
2	64
3	144
4	256

20. Find the slope of the line graphed below.



4-4 The Slope Formula

EXAMPLE

■ **Find the slope of the line described by $2x - 3y = 6$.**

Step 1 Find the x- and y-intercepts.

Let $y = 0$.

$2x - 3(0) = 6$

$2x = 6$

$x = 3$

Let $x = 0$.

$2(0) - 3y = 6$

$-3y = 6$

$y = -2$

The line contains $(3, 0)$ and $(0, -2)$.

Step 2 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{0 - 3} = \frac{-2}{-3} = \frac{2}{3}$$

EXERCISES

Find the slope of the line described by each equation.

21. $4x + 3y = 24$

22. $y = -3x + 6$

23. $x + 2y = 10$

24. $3x = y + 3$

25. $y + 2 = 7x$

26. $16x = 4y + 1$

Find the slope of the line that contains each pair of points.

27. $(1, 2)$ and $(2, -3)$

28. $(4, -2)$ and $(-5, 7)$

29. $(-3, -6)$ and $(4, 1)$

30. $\left(\frac{1}{2}, 2\right)$ and $\left(\frac{3}{4}, \frac{5}{2}\right)$

31. $(2, 2)$ and $(2, 7)$

32. $(1, -3)$ and $(5, -3)$

Answers

13. x-int.: 2; y-int.: -4

14. x-int.: 5; y-int.: 6

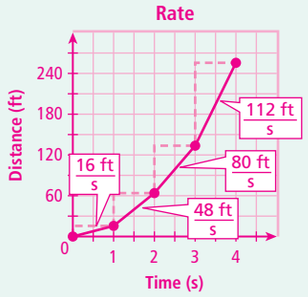
15. x-int.: 3; y-int.: -9

16. x-int.: $-\frac{1}{2}$; y-int.: 1

17. x-int.: -18; y-int.: 3

18. x-int.: $\frac{1}{3}$; y-int.: $-\frac{1}{4}$

19.



20. 5

21. $-\frac{4}{3}$

22. -3

23. $-\frac{1}{2}$

24. 3

25. 7

26. 4

27. -5

28. -1

29. 1

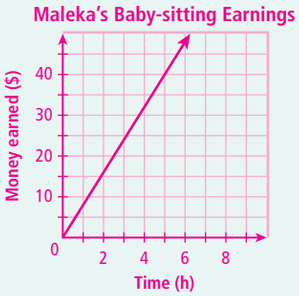
30. 2

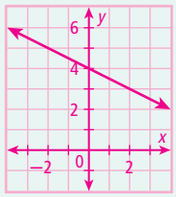
31. undefined

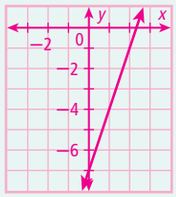
32. 0

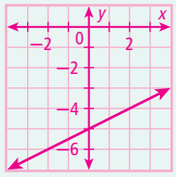
Answers

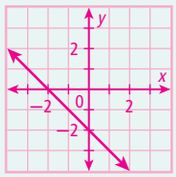
33. yes; -6
34. yes; 1
35. no
36. yes; $-\frac{1}{2}$
37. -12
38. $y = 8x$



39.


40.


41. $y = \frac{1}{3}x + 5$
42. $y = 4x - 9$
43.


44.


45. $y = 2x + 1$
46. $y = -5x - 26$
47. $y = 2x + 2$
48. $y = 2x + 8$

4-5 Direct Variation

EXAMPLE

■ **Tell whether $6x = -4y$ represents a direct variation. If so, identify the constant of variation.**

$$6x = -4y$$
$$\frac{6x}{-4} = \frac{-4y}{-4} \quad \textit{Solve the equation for y.}$$
$$-\frac{6}{4}x = y$$
$$y = -\frac{3}{2}x \quad \textit{Simplify.}$$

This equation represents a direct variation because it can be written in the form $y = kx$, where $k = -\frac{3}{2}$.

EXERCISES

Tell whether each equation is a direct variation. If so, identify the constant of variation.

33. $y = -6x$ **34.** $x - y = 0$

35. $y + 4x = 3$ **36.** $2x = -4y$

37. The value of y varies directly with x, and $y = -8$ when $x = 2$. Find y when $x = 3$.

38. Maleka charges \$8 per hour for baby-sitting. The amount of money she makes varies directly with the number of hours she baby-sits. Write a direct variation equation for the amount y Maleka earns for baby-sitting x hours. Then graph.

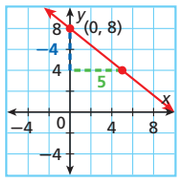
4-6 Slope-Intercept Form

EXAMPLE

■ **Graph the line with slope $= -\frac{4}{5}$ and y-intercept $= 8$.**

Step 1 Plot $(0, 8)$.

Step 2 For a slope of $\frac{-4}{5}$, count **4 down** and **5 right** from $(0, 8)$. Plot another point.



Step 3 Connect the two points with a line.

EXERCISES

Graph each line given the slope and y-intercept.

39. slope $= -\frac{1}{2}$; y-intercept $= 4$

40. slope $= 3$; y-intercept $= -7$

Write the equation that describes each line in slope-intercept form.

41. slope $= \frac{1}{3}$, y-intercept $= 5$

42. slope $= 4$, $(1, -5)$ is on the line.

4-7 Point-Slope Form

EXAMPLE

■ **Write an equation in slope-intercept form for the line through $(4, -1)$ and $(-2, 8)$.**

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{-2 - 4} = \frac{9}{-6} = -\frac{3}{2}$$

Step 2 Write the point-slope form.

$$y - 8 = -\frac{3}{2}[x - (-2)] \quad \textit{Substitute into the point-slope form.}$$
$$y - 8 = -\frac{3}{2}(x + 2)$$

Step 3 Write the slope-intercept form.

$$y = -\frac{3}{2}x + 5 \quad \textit{Solve for y.}$$

EXERCISES

Graph the line described by each equation.

43. $y + 3 = \frac{1}{2}(x - 4)$ **44.** $y - 1 = -(x + 3)$

Write the equation that describes each line in slope-intercept form.

45. slope $= 2$, $(1, 3)$ is on the line.

46. slope $= -5$, $(-6, 4)$ is on the line.

47. $(1, 4)$ and $(3, 8)$ are on the line.

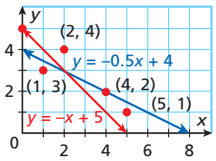
48. $(-2, 4)$ and $(-1, 6)$ are on the line.

4-8 Line of Best Fit

EXAMPLE

■ **Two lines of fit for the data in the table are $y = -x + 5$ and $y = -0.5x + 4$. For each line, find the sum of the squares of the residuals. Which line is a better fit?**

x	1	2	4	5
y	3	4	2	1



$y = -x + 5$:

$1 + 1 + 1 + 1 = 4$

$y = -0.5x + 4$:

$\frac{1}{4} + 1 + 0 + \frac{1}{4} = 2\frac{1}{2}$

$y = -0.5x + 4$ is a better fit.

EXERCISES

49. Two lines of fit for the data in the table are $y = 0.5x$ and $y = x - 1$. For each line, find the sum of the squares of the residuals. Which line is a better fit?

x	1	2	4	5
y	2	0	1	3

50. The lengths and weights of 6 koi in a pond are shown in the table. Find an equation of a line of best fit. How well does the line fit the data?

Length (in.)	9	12	11	15	8	10
Weight (oz)	5	11	9	20	4	7

4-9 Slopes of Parallel and Perpendicular Lines

EXAMPLE

■ **Write an equation in slope-intercept form for the line that passes through $(4, -2)$ and is perpendicular to the line described by $y = -4x + 3$.**

The slope of $y = -4x + 3$ is -4.

The perpendicular line has a slope of $\frac{1}{4}$ and contains $(4, -2)$.

$y + 2 = \frac{1}{4}(x - 4)$ *Substitute into the point-slope form.*

$y = \frac{1}{4}x - 3$ *Solve for y.*

EXERCISES

Identify which lines are parallel.

51. $y = -\frac{1}{3}x$; $y = 3x + 2$; $y = -\frac{1}{3}x - 6$; $y = 3$

52. $y - 2 = -4(x - 1)$; $y = 4x - 4$; $y = \frac{1}{4}x$; $y = -4x - 2$

Identify which lines are perpendicular.

53. $y - 1 = -5(x - 6)$; $y = \frac{1}{5}x + 2$; $y = 5$; $y = 5x + 8$

54. $y = 2x$; $y - 2 = 3(x + 1)$; $y = \frac{2}{3}x - 4$; $y = -\frac{1}{3}x$

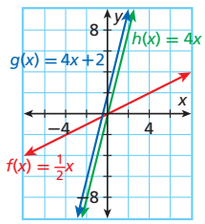
55. Write an equation in slope-intercept form for the line that passes through $(1, -1)$ and is parallel to the line described by $y = 2x - 4$.

4-10 Transforming Linear Functions

EXAMPLE

■ **Graph $f(x) = \frac{1}{2}x$ and $g(x) = 4x + 2$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.**

- Multiply $f(x) = \frac{1}{2}x$ by 8 to get $h(x) = 4x$. This rotates the graph about $(0, 0)$, making it steeper.
- Then add 2 to $h(x) = 4x$ to get $g(x) = 4x + 2$. This translates the graph 2 units up.



EXERCISES

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

56. $f(x) = x$, $g(x) = x + 4$

57. $f(x) = 4x$, $g(x) = -4x$

58. $f(x) = \frac{1}{3}x - 2$, $g(x) = -\frac{1}{3}x - 2$

59. The entrance fee at a carnival is \$3 and each ride costs \$1. The total cost for x rides is $f(x) = x + 3$. How will the graph of this function change if the entrance fee is increased to \$5? if the cost per ride is increased to \$2?

Answers

49. $y = -0.5x$: 4.5, $y = x - 1$: 6.25, $y = 0.5x$ is the better fit.

50. $y \approx 2.3x - 16$; extremely well ($r \approx 0.99$)

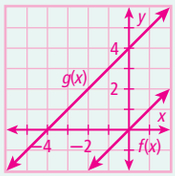
51. $y = -\frac{1}{3}x$ and $y = -\frac{1}{3}x - 6$

52. $y - 2 = -4(x - 1)$ and $y = -4x - 2$

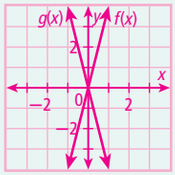
53. $y - 1 = -5(x - 6)$ and $y = \frac{1}{5}x + 2$

54. $y - 2 = 3(x + 1)$ and $y = -\frac{1}{3}x$

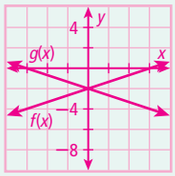
55. $y = 2x - 3$

56. 

trans. 4 units up

57. 

ref. across y-axis

58. 

ref. across y-axis

59. trans. 2 units up; rot. about $(0, 3)$ (steeper)

CHAPTER 4

CHAPTER TEST

Organizer

Objective: Assess students' mastery of concepts and skills in this chapter.

Online Edition

Resources

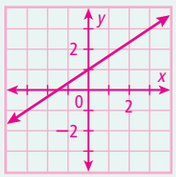
Assessment Resources

Chapter Tests

- Free Response (Levels A, B, C)
- Multiple Choice (Levels A, B, C)
- Performance Assessment

Answers

1.



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State Resources Online

CHAPTER 4

CHAPTER TEST

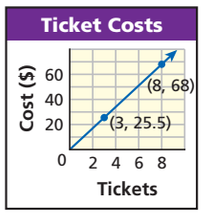
Tell whether each function is linear. If so, graph the function.

1. $3y = 2x + 3$ **yes**
2. $y = x(4 + x)$ **no**
3. Lily plans to volunteer at the tutoring center for 45 hours. She can tutor 3 hours per week. The function $f(x) = 45 - 3x$ gives the number of hours she will have left to tutor after x weeks. Graph the function and find its intercepts. What does each intercept represent?
4. The table shows the number of guppies in an aquarium over time. Graph the data and show the rates of change.

Time (mo)	0	1	2	5	8
Guppies	4	4	10	25	31

Find the slope of each line. Then tell what the slope represents.

5.



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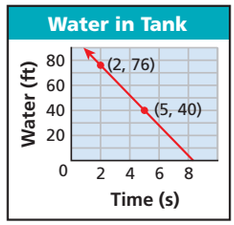

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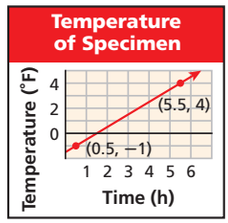


8. A space shuttle travels at a speed of about 5 miles per second. Write a direct variation equation for the distance y the space shuttle will travel in x seconds. Then graph. $y = 5x$
9. Write the equation $2x - 2y = 4$ in slope-intercept form. Then graph the line described by the equation. $y = x - 2$

Write the equation that describes each line in slope-intercept form.

10. slope = 4, (−3, 3) is on the line. $y = 4x + 15$
11. x-intercept = 3, y-intercept = −3 $y = x - 3$

Write an equation in point-slope form for the line with the given slope that contains the given point.

12. slope = −1; (1, 3) $y - 3 = -(x - 1)$
13. slope = 5; (−3, 2) $y - 2 = 5(x + 3)$
14. Four friends recorded the numbers of CDs and video games their families purchased in the last month, as shown in the table. Find an equation of a line of best fit. How well does the line fit the data?

CDs	2	3	5	6
Games	2	5	4	6

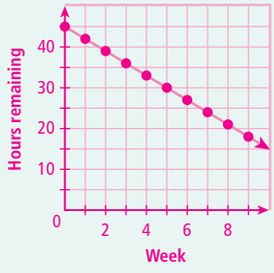
$y \approx 0.7x + 1.45$; moderately well ($r \approx 0.75$)

15. Write an equation in slope-intercept form for the line that passes through (0, 6) and is parallel to the line described by $y = 2x + 3$. $y = 2x + 6$

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

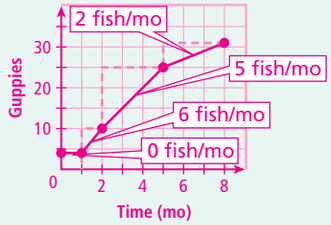
16. $f(x) = 8x$, $g(x) = 4x$
17. $f(x) = -x + 2$, $g(x) = -x - 1$
18. $f(x) = 3x$, $g(x) = 6x - 1$

3.



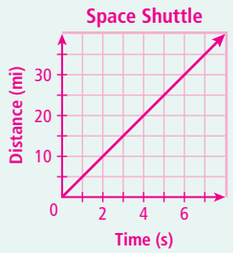
x-int.: 15; *y*-int.: 45; *x*-int.: number of weeks that will have passed when Lily has no vol. hours remaining; *y*-int.: orig. number of vol. hours

4.



5. slope = 8.5; $8.50/ticket
6. slope = −12; height decreases by 12 ft/s
7. slope = 1; temperature increases by 1 °F/h

8.



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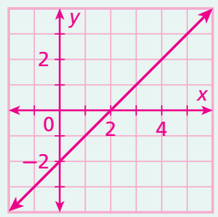


16–18. See Additional Answers.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 4

FOCUS ON SAT

SAT scores are based on the total number of items answered correctly minus a fraction of the number of multiple-choice questions answered incorrectly. No points are subtracted for questions unanswered.

HOT TIP! On the SAT, there is a penalty for incorrect answers on multiple-choice items. Guess only when you can eliminate at least one of the answer choices.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

1. The line through $A(1, -3)$ and $B(-2, d)$ has slope -2. What is the value of d?

(A) $-\frac{3}{2}$

(B) -1

(C) $\frac{1}{2}$

(D) 3

(E) 5

2. The ordered pairs $\{(0, -3), (4, -1), (6, 0), (10, 2)\}$ satisfy a pattern. Which is NOT true?

(A) The pattern is linear.

(B) The pattern can be described by $2x - 4y = 12$.

(C) The ordered pairs lie on a line.

(D) $(-4, 1)$ satisfies the same pattern.

(E) The set of ordered pairs is a function.

3. If y varies directly as x, what is the value of x when $y = 72$?

x	7	12	
y	28	48	72

(A) 17

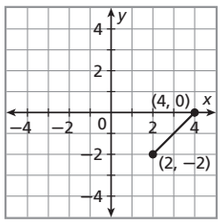
(B) 18

(C) 24

(D) 28

(E) 36

4. The line segment between the points $(4, 0)$ and $(2, -2)$ forms one side of a rectangle. Which of the following coordinates could determine another vertex of that rectangle?



(A) $(-2, 6)$

(B) $(-2, -2)$

(C) $(0, 6)$

(D) $(1, 2)$

(E) $(4, 6)$

5. Which of the following has the same slope as the line described by $2x - 3y = 3$?

(A) $3x - 2y = 2$

(B) $\frac{2}{3}x - y = -2$

(C) $2x - 2y = 3$

(D) $\frac{1}{3}x - 2y = -2$

(E) $-2x - 3y = 2$

TEST PREP DOCTOR

1. Students who chose **A** may have inverted the slope formula. Students who chose **E** most likely used addition in the slope formula instead of subtraction.

2. Students who did not choose **D** should review terminology and properties of linear functions.

3. Students who chose **A** most likely assumed that the difference between x-values was constant. Remind students to look for a value of k so that the given values satisfy $y = kx$.

4. Students who did not choose **A** may not have verified that the line segments forming adjacent sides were perpendicular. Remind students that a rectangle has four right angles.

5. Students who chose **C** selected a function with the same x-coefficient as the given function, but not the same slope. Students who chose **A** selected an answer with the same y-intercept as the given function.

Organizer

Objective: Provide practice for college entrance exams such as the SAT.

INTERACTIVE **Online Edition**

Resources

College Entrance Exam Practice

Questions on the SAT represent the following math content areas:

Number and Operations, 20–25%

Algebra and Functions, 35–40%

Geometry and Measurement, 25–30%

Data Analysis, Statistics, and Probability, 10–15%

Items on this page focus on:

- Algebra and Functions
- Geometry and Measurement

CHAPTER 4 TEST TACKLER

Organizer

Objective: Provide opportunities to learn and practice common test-taking strategies.

Online Edition

This Test Tackler focuses on how to recognize the distracters given in multiple-choice test items. Tell students that the "wrong" answer choices given in multiple-choice items are not randomly written, but are created based on common student errors.

CHAPTER 4 TEST TACKLER

Standardized Test Strategies

Multiple Choice: Recognize Distracters

In multiple-choice items, the options that are incorrect are called *distracters*. This is an appropriate name, because these incorrect options can distract you from the correct answer.

Test writers create distracters by using common student errors. Beware! Even if the answer you get when you work the problem is one of the options, it may not be the correct answer.

EXAMPLE 1

What is the y-intercept of $4x + 10 = -2y$?

Ⓐ 10 Ⓒ -2.5

Ⓑ 5 Ⓓ -5

Look at each option carefully.

Ⓐ This is a distracter. The y-intercept would be 10 if the function was $4x + 10 = y$. A common error is to ignore the coefficient of y.

Ⓑ This is a distracter. Another common error is to divide by 2 instead of -2 when solving for y.

Ⓒ This is a distracter. One of the most common errors students make is confusing the x-intercept and the y-intercept. This distracter is actually the *x-intercept* of the given line.

Ⓓ This is the correct answer.

EXAMPLE 2

What is the equation of a line with a slope of -4 that contains $(2, -3)$?

Ⓕ $y - 3 = -4(x - 2)$ Ⓗ $y + 3 = -4(x - 2)$

Ⓖ $y - 2 = -4(x + 3)$ Ⓙ $y + 4 = -3(x - 2)$

Look at each option carefully.

Ⓕ This is a distracter. Students often make errors with positive and negative signs. You would get this answer if you simplified $y - (-3)$ as $y - 3$.

Ⓖ This is a distracter. You would get this answer if you switched the x-coordinate and the y-coordinate.

Ⓗ This is the correct answer.

Ⓙ This is a distracter. You would get this answer if you substituted the given values incorrectly in the point-slope equation.

HOT TIP! When you calculate an answer to a multiple-choice test item, try to solve the problem again with a different method to make sure your answer is correct.

Read each test item and answer the questions that follow.

Item A

A line contains $(1, 2)$ and $(-2, 14)$. What are the slope and y-intercept?

Ⓐ Slope $= -4$; y-intercept $= -2$

Ⓑ Slope $= 4$; y-intercept $= 6$

Ⓒ Slope $= -\frac{1}{4}$; y-intercept $= 1$

Ⓓ Slope $= -4$; y-intercept $= 6$

1. What common error does the slope in choice B represent?
2. The slope given in choice A is correct, but the y-intercept is not. What error was made when finding the y-intercept?
3. What formula can you use to find the slope of a line? How was this formula used incorrectly to get the slope in choice C?

Item B

Which of these functions has a graph that is NOT parallel to the line described by $y = \frac{1}{2}x + 4$?

Ⓕ $y = 6 - \frac{1}{2}x$

Ⓖ $y = \frac{1}{2}x + 6$

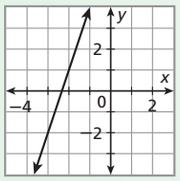
Ⓗ $-2y = -x + 1$

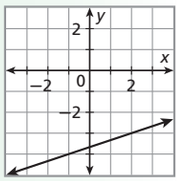
Ⓙ $2y = x$

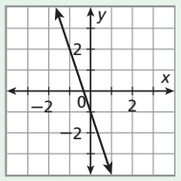
4. When given two linear functions, describe how to determine whether their graphs are parallel.
5. Which is the correct answer? Describe the errors a student might make to get each of the distracters.

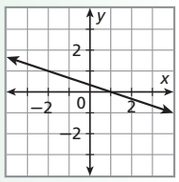
Item C

Which of these lines has a slope of -3?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

6. Which two answer choices can be eliminated immediately? Why?
7. Decribe how to find the slope of a line from its graph.
8. What common error does choice A represent?
9. What common error does choice D represent?
10. Which is the correct answer?

Item D

Which is NOT a linear function?

Ⓕ $f(x) = 4 + x$

Ⓖ $f(x) = -x - 4$

Ⓗ $f(x) = 4x^2$

Ⓙ $f(x) = \frac{1}{4}x$

11. When given a function rule, how can you tell if the function is linear?
12. What part of the function given in choice G might make someone think it is not linear?
13. What part of the function given in choice J might make someone think it is not linear?
14. What part of the function given in choice H makes it NOT linear?

TEST PREP DOCTOR Show students how common errors, such as dropping negative signs or switching the x- and y-coordinates, will lead to an incorrect answer. Point out instances in which that incorrect answer is given as one of the answer choices. Any error analysis that students can do on a regular basis will help them to recognize distracters. Use **Problems 1, 9,** and **10** to discuss common errors with linear functions.

Answers

1. The slope should be -4 instead of 4. An error was made with the neg. sign.
2. Possible answer: When $y = mx + b$ was solved for b using the point $(1, 2)$, 4 was subtracted from both sides instead of -4.
3. $m = \frac{y_2 - y_1}{x_2 - x_1}$; the formula was incorrectly inverted.
4. They have the same slope.
5. **F** is the correct answer because it is the only option whose slope is not equal to $\frac{1}{2}$.
   Possible answers: You might get **G** or **J** if you do not read the question correctly and think you were looking for a line par. to $y = \frac{1}{2}x + 4$. You might get **H** if you divide by 2 instead of -2 when solving for y.
6. **A** and **C**; a line with a negative slope falls from left to right, so the correct answer cannot be **A** or **C**.
7. Possible answer: Locate 2 points on the graph. Begin at one of the points and count the rise and the run to the second point. Divide the rise by the run.
8. The slope of the line in **A** is 3. The line increases from left to right. The question asked for a slope of -3.
9. The slope of the line in **D** is $-\frac{1}{3}$, not -3. The rise is -1 and the run is 3.
10. B
11. When a linear equation is written in standard form, x and y both have exponents of 1, x and y are not multiplied together, and x and y do not appear in denominators or in exponents.
12. the negative sign
13. the fraction
14. x has an exponent of 2.

Answers to Test Items

A. D

B. F

C. B

D. H

Learn It Online
State Resources Online

Organizer

Objective: Provide review and practice for this chapter and standardized tests.

Online Edition

Resources

Assessment Resources

Chapter Cumulative Test

Learn It Online
State Test Practice

Learn It Online
State Resources Online

CHAPTER 4 STANDARDIZED TEST PREP

Learn It Online
State Test Practice

CUMULATIVE ASSESSMENT

Multiple Choice

1. What is the value of $2 - [1 - (2 - 1)]$?

Ⓐ -2
Ⓑ 0
Ⓒ 2
Ⓓ 4

2. Frank borrowed \$5000 with an annual simple interest rate. The amount of interest he owed after 6 months was \$300. What is the interest rate of the loan?

Ⓕ 1%
Ⓖ 6%
Ⓗ 10%
Ⓙ 12%

3. Patty's Pizza charges \$5.50 for a large pizza plus \$0.30 for each topping. Pizza Town charges \$5.00 for a large pizza plus \$0.40 for each topping. Which inequality can you use to find the number of toppings x so that the cost of a pizza at Pizza Town is greater than the cost of a pizza at Patty's Pizza?

Ⓐ $(5 + 0.4)x > (5.5 + 0.3)x$
Ⓑ $5.5x + 0.3 > 5x + 0.4$
Ⓒ $5.5 + 0.3x > 5 + 0.4x$
Ⓓ $5 + 0.4x > 5.5 + 0.3x$

4. The side length of a square s can be determined by the formula $s = \sqrt{A}$ where A represents the area of the square. What is the side length of a square with area 0.09 square meter?

Ⓕ 0.0081 meters
Ⓖ 0.81 meters
Ⓗ 0.03 meters
Ⓙ 0.3 meters

5. What is the value of $f(x) = -3 - x$ when $x = -7$?

Ⓐ -10
Ⓑ -4
Ⓒ 4
Ⓓ 10

6. Which relationship is a direct variation?

Ⓕ

x	1	2	3	4
y	-1	0	1	2

Ⓖ

x	1	2	3	4
y	0	-1	-2	-3

Ⓗ

x	1	2	3	4
y	3	5	7	9

Ⓙ

x	1	2	3	4
y	3	6	9	12

7. Which function has x-intercept -2 and y-intercept 4?

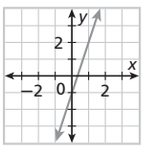
Ⓐ $2x - y = 4$
Ⓑ $2y - x = 4$
Ⓒ $y - 2x = 4$
Ⓓ $x - 2y = 4$

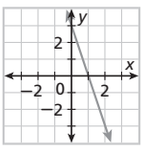
8. Which equation describes the relationship between x and y in the table below?

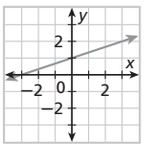
x	-8	-4	0	4	8
y	2	1	0	-1	-2

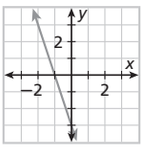
Ⓕ $y = -4x$
Ⓖ $y = -\frac{1}{4}x$
Ⓗ $y = 4x$
Ⓙ $y = \frac{1}{4}x$

9. Which graph is described by $x - 3y = -3$?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

Answers

1. C
2. J
3. D
4. J
5. C
6. J
7. C
8. G
9. B
10. G
11. C
12. H
13. C
14. 10.5
15. 7.5
16. 14

When answering multiple-choice test items, check that the test item number matches the number on your answer sheet, especially if you skip test items that you plan to come back to.

10. Which steps could you use to graph the line that has slope 2 and contains the point $(-1, 3)$?

Ⓕ Plot $(-1, 3)$. Move 1 unit up and 2 units right and plot another point.

Ⓖ Plot $(-1, 3)$. Move 2 units up and 1 unit right and plot another point.

Ⓗ Plot $(-1, 3)$. Move 1 unit up and 2 units left and plot another point.

Ⓙ Plot $(-1, 3)$. Move 2 units up and 1 unit left and plot another point.

11. Which line is parallel to the line described by $2x + 3y = 6$?

Ⓐ $3x + 2y = 6$ Ⓒ $2x + 3y = -6$

Ⓑ $3x - 2y = -6$ Ⓓ $2x - 3y = 6$

12. Which function's graph is NOT perpendicular to the line described by $4x - y = -2$?

Ⓕ $y + \frac{1}{4}x = 0$ Ⓗ $3y = \frac{3}{4}x + 3$

Ⓖ $\frac{1}{2}x = 10 - 2y$ Ⓙ $y = -\frac{1}{4}x + \frac{3}{2}$

13. Company A charges \$30 plus \$0.40 per mile for a car rental. The total charge for m miles is given by $f(m) = 30 + 0.4m$. For a similar car, company B charges \$30 plus \$0.30 per mile. The total charge for m miles is given by $g(m) = 30 + 0.3m$. Which best describes the transformation from the graph of $f(m)$ to the graph of $g(m)$?

Ⓐ Translation up

Ⓑ Translation down

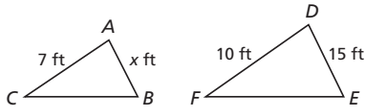
Ⓒ Rotation

Ⓓ Reflection

Gridded Response

14. What is the value of x in the diagram below?

$\triangle ABC \sim \triangle DEF$



15. What is the 46th term in the arithmetic sequence $-1.5, -1.3, -1.1, -0.9, \ldots$?

16. What is the y-intercept of $y - 2 = 3(x + 4)$?

Short Response

17. A video store charges a \$10 membership fee plus \$2 for each movie rental. The total cost for x movie rentals is given by $f(x) = 2x + 10$.

a. Graph this function.

b. Give a reasonable domain and range.

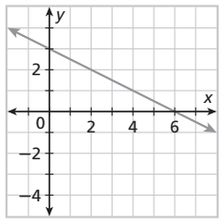
18. The table below shows the federal minimum wage in different years.

Year	1960	1970	1980	1990	2000
Minimum Wage (\$)	1.00	1.60	3.10	3.80	5.15

a. Find the rate of change for each ten-year time period. Show your work.

b. During which time period did the minimum wage increase the fastest? Explain what the rate of change for this time period means.

19. a. Find the slope of the line below.



b. Write an equation in slope-intercept form for a line that is perpendicular to the line in part **a** and has the same y-intercept as the function in part **a.** Show your work and explain how you found your answer.

Extended Response

20. The relationship between the wind speed at a given temperature and the apparent temperature is approximately linear. The table shows the apparent temperature at various wind speeds when the actual temperature is t °F.

Wind Speed (mi/h)	5	10	15
Apparent Temperature (°F)	36	34	32

a. Write an equation in slope-intercept form that describes the data in the table. Explain how you found your answer.

b. What does the slope mean in this situation?

c. What is the actual temperature t? Explain.

d. Determine the apparent temperature when the wind speed is 12 miles per hour. Show your work.

Short-Response Rubric

Items 17–19

2 Points = The student's answer is an accurate and complete execution of the task or tasks.

1 Point = The student's answer contains attributes of an appropriate response but is flawed.

0 Points = The student's answer contains no attributes of an appropriate response.

Extended-Response Rubric

Item 20

4 Points = The student writes the correct equation and explanation and shows all work in part **a,** gives the correct explanation for the slope in part **b,** gives the correct answer and explanation in part **c,** and determines the correct answer and shows all work in part **d.**

3 Points = The student writes the correct equation and explanation but does not show all work in part **a,** gives the correct explanation for the slope in part **b,** gives the correct answer and some explanation in part **c,** and determines the correct answer and shows some work in part **d.**

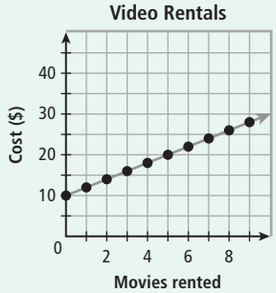
2 Points = The student writes the correct equation and explanation but shows no work in part **a,** gives the correct explanation for the slope in part **b,** gives the correct answer but no explanation in part **c,** and determines the correct answer and shows some work in part **d;** or the student attempts to answer all parts and correctly answers only two parts.

1 Point = The student writes the correct equation but gives no explanation and shows no work in part **a,** gives the correct explanation for the slope in part **b,** gives the correct answer but no explanation in part **c,** and determines the correct answer but shows no work in part **d;** or the student attempts to answer all parts of the problem but does not correctly answer any part.

0 Points = The student does not answer correctly and does not attempt all parts of the problem.

Answers

17a.



b. D: whole numbers; R: even whole numbers greater than or equal to 10

18a. 1960 to 1970: 0.06; 1970 to 1980: 0.15; 1980 to 1990: 0.07; 1990 to 2000: 0.135

b. 1970 to 1980; on average, min. wage increased by \$0.15 per year during this time period.

19a. $-\frac{1}{2}$

b. $y = 2x + 3$

20a. $y = -\frac{2}{5}x + 38$

b. the amount by which the apparent temp. decreases each time the wind speed increases by 1 mi/h

c. 38°F

d. 33.2°F